

# Intra- and Inter-temporal Redistribution with Borrowing Constraints and Distortionary Taxation \*

Rong Li<sup>†</sup>, Xiaohui Tian<sup>‡</sup>

## Abstract

In a dynamic general equilibrium model with financial imperfections and heterogeneous agents, we revisit the effects of two types of fiscal policy: intratemporal redistribution; and a debt-financed uniform transfer, which is interpreted as intertemporal redistribution. We find that, under flexible prices, a uniform debt-financed transfer has a positive effect on consumption on impact but not in the long-run; and it causes a persistent contraction of output. Moreover, due to distortionary taxation and borrowing constraints, the uniform transfer leads to heterogeneous impacts and welfare implications on households. A Robin-Hood intratemporal redistribution to low wealthy households is found to be expansionary on private consumption and effective labor hours. In addition, its output multiplier is positive on impact but turns into a negative value in the long run.

JEL Classification: E32, E62, H30, H31

Keywords: public debt, redistribution, transfers, distortionary taxation, incomplete markets, borrowing constraints

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\*We thank Bill Dupor, Paul Evans, and Pok-Sang Lam for advice, Jinhui Bai, Yili Chien, Zhigang Feng, Fernando M. Martin, Alex Monge-Naranjo, Byoung Hoon Seok, and David Wiczer for valuable comments. Part of this article was completed while Rong Li was visiting the Federal Reserve Bank of St. Louis, and their hospitality is gratefully acknowledged.

<sup>†</sup>China Financial Policy Research Center, School of Finance, Renmin University of China, Email: lirong.sf@ruc.edu.cn, Phone: +86(10)82500633

<sup>‡</sup>SARD, Renmin University of China, Email: tianxiaohui@ruc.edu.cn

# 1 Introduction

Since the Great Recession, the impacts of fiscal stimulus programmes have once again turned into central to policy debates. New empirical analyses have adopted varieties of econometric methods to identify exogenous changes in government spending and assess the impacts on output, employment and consumption. (See, for example, Mountford and Uhlig (2009); Fisher and Peters (2010); Monacelli et al. (2010); Ramey (2011); Barro and Redlick (2011); Auerbach and Gorodnichenko (2012); Ilzetzi et al. (2013); Nakamura and Steinsson (2014); Ramey and Zubairy (2014); Dupor and Li (2015).) On the other hand, a number of recent theoretical studies have focused on the circumstances that could vary the effects of a positive government spending shock in dynamic general equilibrium models. (See, for example, Cogan et al. (2010); Christiano et al. (2011); Woodford (2011); Drautzburg and Uhlig (2011); Davig and Leeper (2011); Corsetti et al. (2012); Erceg and Lindé (2014); Carlstrom et al. (2014).)

However, as pointed out by Oh and Reis (2012) and Bilbiie et al. (2013), most recent work has focused on increases in government purchases, while in the U.S. and many other countries, not only government purchases but also transfers are important components in a fiscal stimulus package. Taking the American Recovery and Reinvestment Act (ARRA) of 2009 as an example, Drautzburg and Uhlig (2011) calculate that transfers account for 55.14% of total stimulus package. Hence, it is important to investigate the aggregate impacts and distributional consequences of transfers. In an influential and inspiring paper, Bilbiie et al. (2013) explore fiscal stimulus policies in the form of temporary transfers. They find that, under flexible prices, revenue-neutral intratemporal redistribution and debt-finance tax cuts are either neutral or display effects that are at odds with the empirical evidence. In this article, we re-examine the effects of these two types of fiscal policy under flexible prices but in a dynamic general equilibrium model that is calibrated to match the U.S. wealth and earnings distributions.

There are a number of important differences between Bilbiie et al. (2013) and our model. Like them, we emphasize public debt and borrowing constraints. In contrast to them, our

model is an Aiyagari (1994) style incomplete markets model with idiosyncratic, uninsurable uncertainty about labor productivity. Hence, the heterogeneous households differ in their wealth and earnings ability instead of their degree of impatience. Second, government in our model is allowed to raise revenues with distortionary taxation. As emphasized by Uhlig (2010), government expenditures are financed eventually with distortionary taxes, creating disincentive effects. Moreover, the income tax in our model is also progressive, so a uniform tax cut or transfer would have distributional impacts among households. Finally, changes in capital accumulation in our model generate long-run consequences under flexible prices.

Our analysis is also related to Oh and Reis (2012), but with some significant differences. In contrast to them, we allow debt-financed fiscal stimulus programmes in our economy and consider both short-run and long-run effects. Government debt has been proved to have a significant role in fiscal policy(See, for example, Leeper et al. (2010); Corsetti et al. (2012); Bilbiie et al. (2013); Kliem and Kriwoluzky (2014); Nickel and Tudyka (2014)). Leeper et al. (2010) and Uhlig (2010) argue that debt-financed fiscal shocks generate long-lasting dynamics; and short-run and long-run fiscal multipliers can differ dramatically. Second, the distortionary and progressive taxation in our model provides a channel such that a uniform fiscal policy, for example a uniform transfer, could have heterogeneous impacts on households. Finally, it is argued that policy should care about welfare and can result in heterogeneous welfare effects across the population. Hence, we also address welfare issues in this paper.

Our model is based on the one in Castaneda et al. (2003). The key ingredients are idiosyncratic, uninsurable uncertainty to labor productivity; altruistic households that go through working-age and retirement stages; a borrowing constraint; a government who collects progressive income taxes to finance its expenditure including goods purchases and social security transfers to retired population. In our policy experiments, fiscal stimulus programmes can be debt-financed and debt is repaid by adjustment in income taxes.

The main results are: a uniform debt-financed transfer leads to a positive response of aggregate consumption on impact but not in the long-run; and it causes a persistent con-

traction of output. Moreover, due to distortionary taxation and borrowing constraints, the uniform transfer leads to heterogeneous impacts and welfare implications on households. In addition, it reduces inequality in terms of welfare between the richest households and others but enlarges inequality within the mid- and low-income classes. Our analysis reveals that an intertemporal transfer affects individual decisions through four channels: (i) income effects brought directly by government transfers; (ii) changes in future income taxes; (iii) the borrowing constraint; and (iv) general equilibrium effects. The uniform transfer brings more income to every household, but the wealthiest group of households bears the brunt of the adjustment in future taxes due to the distortionary and progressive taxation. As a result, rational households in our model economy will exhibit diverse responses to this uniform transfer shock. Moreover, the borrowing constraint further distorts behaviors of the least wealthy households and forces them to work more and consume less in periods of higher income tax rates. Finally, changes in factor prices, i.e. the general equilibrium effects, deliver another channel such that households respond differently to a uniform transfer shock. Since households differ in wealth and labor productivity, the relative importance between labor income and capital income is different across households. Hence, they react in different ways to changes in wage rates and interest rates.

On the other hand, a Robin-Hood intratemporal redistribution from the wealthiest group of households to the least wealthy ones is found to be expansionary on private consumption and effective labor hours. In addition, its output multiplier is positive on impact but turns into a negative value in the long run. In contrast to Bilbiie et al. (2013), the income effects is not symmetric between receivers and payees of the transfer in our experiment. Due to the heterogeneity in households wealth, receivers who are less wealthy have higher marginal propensity to consume, while payees have the ability and are willing to smooth consumption. Therefore, aggregate consumption is stimulated as a result of such intratemporal transfers. On the other hand, as the payees who increase labor supply have higher average labor productivity than the receivers who reduce working hours, total effective labor supply increases

and output is boosted on impact. Finally, aggregate capital drops because payees decrease savings and that causes long run effects on the economy.

The paper is organized as follows. Section 2 provides an overview of the model and calibration. Section 3 discusses the short-run and long-run impacts of a uniform debt-financed tax cut and a Robin-Hood intratemporal redistribution. Section 4 concludes.

## 2 A quantitative model

In this section, We construct a heterogeneous-agent dynamic general equilibrium model based on Castaneda et al. (2003) and calibrate it such that it matches the U.S. distributions of wealth and earnings as well as several other moments of the data.

### 2.1 Labor productivity shocks

The model economy contains a unit mass of continuum of households, who differ in their wealth and labor productivity. A household has two stages in her life: working-age and retired. A working-age household faces an uninsured idiosyncratic stochastic process that determines her labor productivity as well as the transition to retirement. A retired household faces an exogenous probability of dying and will be replaced by a working-age descendant who inherits her asset once she dies. Following Castaneda et al. (2003), a one-dimensional shock,  $s$ , is used to denote the household's random age and labor productivity jointly. We assume that this is an independent and identically distributed process which follows a finite state Markov chain. The conditional transition probabilities are given by  $\Gamma_{s's} = Pr\{s_{t+1} = s' | s_t = s\}$ , where  $s, s' \in S = \{\xi \cup R\}$ .  $\xi = \{\epsilon_l, \epsilon_2, \epsilon_3, \epsilon_h\}$  and  $R = \{0, 0, 0, 0\}$  are two 4-dimensional sets containing the labor productivity of working-age households and retired people, respectively. There are four retirement states, that is because we use the labor productivity in the last period before retirement to keep track of the earnings ability in order to capture the inter-generational transmission of earnings ability. The transition matrix can

be partitioned into four parts:

$$\Gamma_{s's} = \begin{bmatrix} \Gamma_{\epsilon\epsilon} & \Gamma_{\epsilon R} \\ \Gamma_{R\epsilon} & \Gamma_{RR} \end{bmatrix}$$

where,  $\Gamma_{\epsilon\epsilon}$  contains transition probabilities of working-age households that are still of working-age in the next period;  $\Gamma_{\epsilon R} = p_r I$  denotes the transition probabilities from the working-age states into the retirement states, where  $p_r$  is the probability of retiring, and  $I$  is the identity matrix<sup>1</sup>;  $\Gamma_{R\epsilon}$  describes the transition from the retirement states into the working-age states when a retired household dies and is replaced by her descendant<sup>2</sup>;  $\Gamma_{RR} = (1 - p_d)I$ , where  $p_d$  is the probability of dying, denotes the changes in the retirement states of retired households that are still retired in the next period. Following Castaneda et al. (2003),  $\epsilon_l$  is normalized to be one.  $\Gamma_{\epsilon\epsilon}$  and  $\epsilon_2, \epsilon_3, \epsilon_h$  are selected to match the wealth and earnings distributions in U.S. data. Moreover, following Castaneda et al. (2003), two parameters,  $\phi_1$  and  $\phi_2$ , are used to determine the intergenerational persistence of earnings and thus  $\Gamma_{R\epsilon}$ .<sup>3</sup>

## 2.2 Preferences

Households value consumption and leisure, and they are altruistic towards their descendants. Households preferences can be described as:

$$E \sum_{t=0}^{\infty} \beta^t \left[ \frac{c_t^{1-\sigma}}{1-\sigma} + \chi \frac{(\bar{l} - l_t)^{1-\theta}}{1-\theta} \right]$$

where,  $\beta \in (0, 1)$  is the discount factor;  $c_t$  is consumption;  $l_t \in [0, 1]$  is labor supply,  $\bar{l}$  is a fixed endowment of hours in each period.  $\sigma$  is the inverse of intertemporal elasticity of substitution and  $\theta$  captures labor elasticity.  $\chi$  governs utility from leisure.

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<sup>1</sup>This means that every working-age household faces the same probability of retiring.

<sup>2</sup>We further assume that every retired household faces the same probability of dying

<sup>3</sup>See the appendix in Castaneda et al. (2003) for details.

## 2.3 Households problem

Households accumulate wealth  $a_t$  in the form of real capital  $k_t$  and real government debt  $b_t$ , to smooth their streams of consumption against the idiosyncratic shocks to labor productivity as well as aggregate shocks. We further assume that asset holdings belong to a compact set  $\mathbf{A}$ , and the lower bound of this set is a form of borrowing constraint.<sup>4</sup> The production sector is assumed to be perfectly competitive, which implies that factor prices are given by their corresponding marginal productivities.

The individual states are, therefore,  $(a, s)$ . Households choose consumption  $c$ , labor supply  $l$  and savings  $q$  to maximize their utility in an infinite horizon.

The recursive formulation of a household's problem is:

$$v(a, s) = \max_{\{c, q, l\}} \{u(c, l) + \beta E[v(a', s'|s)]\}$$

s.t.

$$c + q = y - \tau(y) + a$$

$$y = ra + wl\epsilon + TrI_{s \in R}$$

$$a' = \begin{cases} q - \tau_E(q) & \text{if } s \in R \text{ and } s' \in \xi \\ q & \text{otherwise} \end{cases}$$

and

$$q = f(a, s) \geq \underline{a}$$

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<sup>4</sup>We use zero as the lower bound. As shown in Huggett (1993), there exists an upper bound for the asset holdings as long as the after-tax rate of return to saving is smaller than the rate of time preference.

where  $y$  is pre-tax income including capital income, labor income which can be earned only by working-age households, and social security income  $TrI_{s \in R}$ , that can be earned only by retired households;  $\tau(y)$  is the progressive income tax;  $q$  is savings choice and  $a'$  is the asset holding in next period, and these two are not always equal because a descendant has to pay estate taxes in order to inherit her parent's asset;  $\tau_E$  is the estate tax, if possible;  $\underline{a}$  is the lower bound of savings;  $f$  is the decision rule for savings; and the real interest rate and wage rate are given by:

$$r = \alpha \left(\frac{K}{L}\right)^{\alpha-1} - \delta, \quad w = (1 - \alpha) \left(\frac{K}{L}\right)^{\alpha}$$

where  $\alpha$  is capital share in the production function;  $K$  is aggregate capital;  $L$  is the aggregate effective labor.

## 2.4 Taxes

This subsection describes the income and estate tax functions. The income tax function is taken from Gouveia and Strauss (1994) and Castaneda et al. (2003):

$$\tau(y) = a_0[y - (y^{-a_1} + a_2)^{-1/a_1}] + a_3y$$

The proportionate part of the income tax,  $a_3$ , serves as a policy instrument. The government in our model economy adjusts this variable to stabilize public debt.

For the estate tax function, there is a lower bound for this tax,  $\underline{q}$ . Bequest that below this level will not be charged any tax and that exceeds this level is subject to a proportionate estate tax rate  $\tau_E$ .

$$\tau_E(q) = \begin{cases} 0 & \text{for } q < \underline{q} \\ \tau_E(q - \underline{q}) & \text{for } q > \underline{q} \end{cases}$$



## 2.5 Government

The government in this model purchases goods, pays social security transfers to retired households and collects income taxes and estate taxes to finance its expenditure. There exists real public debt that pays the same real interest rate as capital does.

The government budget constraint is

$$G + (1 + r)B + \int Tr I_{s \in R} \Gamma = \int \tau(y) \Gamma + \int \tau_E \Gamma + B'$$

where,  $G$  is government purchases;  $Tr$  is the amount of social security transfer;  $\tau(y)$  and  $\tau_E$  are income and estate taxes, respectively;  $B$  and  $B'$  are current and next period government debt, respectively;  $\Gamma$  is the households distribution on  $(a, s)$ . We assume a constant level of government purchases.

## 2.6 Aggregation and markets clearing

The aggregate capital satisfies:

$$K = \int a d\Gamma - B$$

That is, aggregate capital equals to total private savings minus real government debt.

Labor market clearing requires:

$$L = \int l \epsilon d\Gamma$$

Private sector goods are used as households consumption, investment, and government goods consumption:

$$Y = \int c \Gamma + K' - (1 - \delta)K + G$$

## 2.7 Stationary Equilibrium

The aggregate state is a measure of households,  $\Gamma$ , defined over a family of subsets of  $\{S \times \mathbf{A}\}$ . The stationary equilibrium requires that the measure of households remains invariant,

although each individual household may change her position in the households distribution from one period to the next. A stationary equilibrium is then a measure  $\Gamma$ , a pair of individual functions  $v$  and  $f$ , pricing functions  $r$  and  $w$ , and government policies  $G$  and  $B$ , such that (i)  $(v, f)$  solves the household’s problem, (ii)  $(r, w)$  are competitive, (iii)  $\Gamma$  is stationary, (iv)  $G$  and  $B$  solves the government budget constraint<sup>5</sup>, and (v) markets clear.

## 2.8 Calibration

We adopt the parameters governing the joint age and labor productivity process and households utility directly from Castaneda et al. (2003) and calibrate other parameters such that the stationary equilibrium can match the empirical moments in the data. The duration of each time period is one year in the model.

**Utility function and production technology:** We set  $\sigma = 1.5, \theta = 1.016, \chi = 1.138, \bar{l} = 3.2$  which are the same as those used in Castaneda et al. (2003). We set  $\beta = 0.9268$  such that the aggregate capital to output ratio in the stationary equilibrium is 3.13. The capital depreciation rate  $\delta = 0.059$  is taken from Castaneda et al. (2003) to target the steady state annual interest rate. The capital income share  $\alpha$  is set to 0.376.

**The joint age and labor productivity process:** Castaneda et al. (2003) calibrate this process to match the wealth and earnings distributions in U.S. data. Since there is an update regarding those distributions<sup>6</sup>, we follow the strategy in Castaneda et al. (2003) and slightly adapt their parameters to match the updated data. The probability of retirement is set to 0.0222 implying an average working duration of 45 years. The probability of dying is 0.055 indicating an average life of retirement of 18 years. The set of labor productivity parameters is set to be  $\epsilon = \{\epsilon_L = 1.0, \epsilon_2 = 3.15, \epsilon_3 = 9.78, \epsilon_H = 1061\}$ . Table 1 displays the transition probabilities of the process on the labor productivity for working-age households that remain of working-age one period later,  $\Gamma_{\epsilon\epsilon}$ . All rows sum up to 97.78% because that a worker has a probability of 2.22% to be retired. This table illustrates that the labor productivity

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<sup>5</sup>We assume, in steady state, government debt is zero.

<sup>6</sup>See Díaz-Giménez et al. (2011).

shocks are persistent. A household whose current productivity is  $\epsilon_L$  is most likely to make a transition to  $\epsilon_2$  than to any of the other levels. Households with productivity  $\epsilon_2$  or  $\epsilon_3$  are most likely to move to  $\epsilon_L$ . It is very hard for a household to move from any other state to  $\epsilon_H$ , and when a household draws a productivity of  $\epsilon_H$ , it is highly possible that it will draw back to  $\epsilon_L$  in the near future. Parameters that governs the intergeneration of transmission  $\phi_1$  and  $\phi_2$  are taken from Castaneda et al. (2003) and set to 0.969 and 0.525, respectively.

Table 1:  $\Gamma_{\epsilon\epsilon}(\%)$

From $s$	To $s'$			
	$s' = \epsilon_L$	$s' = \epsilon_2$	$s' = \epsilon_3$	$s' = \epsilon_H$
$s = \epsilon_L$	96.241	1.14	0.39	0.006
$s = \epsilon_2$	3.07	94.33	0.37	0.000
$s = \epsilon_3$	1.50	0.43	95.82	0.02
$s = \epsilon_H$	10.66	0.49	6.13	80.51

The above calibration procedure implies the following stationary distribution of working-age households. Table 2 shows that over 61% of working-age households are of type  $s = \epsilon_L$ . The invariant masses of households of type  $s = \epsilon_2$  and  $s = \epsilon_3$  are 22.3% and 16.54%, respectively. Finally, very few working-age households have the highest labor productivity.

Table 2: Stationary distribution of working-age households  $\gamma_\epsilon^*(\%)$ , and relative labor productivity  $e(s)$

	$s = \epsilon_L$	$s = \epsilon_2$	$s = \epsilon_3$	$s = \epsilon_H$
$e(s)$	1.00	3.15	9.78	1061.00
$\gamma_\epsilon^*(\%)$	61.12	22.30	16.54	0.04

**Government sector parameters:** Parameters of the effective income tax function are taken from Gouveia and Strauss (1994):  $a_0 = 0.258$ ,  $a_1 = 0.768$ ;  $a_2 = 0.491$  is taken from Castaneda et al. (2003); while  $a_3$  is set to 0.144 in order to balance the government budget such that government debt in stationary equilibrium is zero.  $G_{ss} = 0.8450$  is chosen

such that the steady state total government spending is 21.5% of GDP<sup>7</sup>. Social security transfer  $Tr = 0.5255$  is selected to match the target: total transfers equal to 4.9% of output.  $\tau_E = 0.0028$  and  $\underline{z} = 35.2$  are selected such that tax exempt is about ten times of average GDP and estate tax revenue is about 0.2 percent of GDP.

Table 3: Aggregate statistics

	$\frac{K}{Y}$	$\frac{G}{Y}$	$\frac{I}{Y}$	$\frac{\tau_E}{Y}$	$\frac{Tr}{Y}$	$\frac{labor}{I}$
target	3.13	0.215	0.186	0.002	0.049	0.33
model	3.13	0.240	0.185	0.002	0.043	0.34

Table 3 and 4 display the steady state aggregate statistics and the wealth and earnings distributions of the model and data, respectively. The wealth and earnings distributions are from Díaz-Giménez et al. (2011). The model matches aggregate moments and the wealth and earnings distributions reasonably well.

Table 4: Wealth and Earnings distributions (%)

	Quintiles					Top(%)		
	1st	2nd	3rd	4th	5th	90-95	95-99	99-100
Distribution of Wealth								
data	-0.2	1.1	4.5	11.2	83.4	11.1	26.7	33.6
model	0.1	1.0	1.5	14.3	83.1	17.3	17.9	31.2
Distribution of Earnings								
data	-0.1	4.2	11.7	20.8	63.5	11.7	16.6	18.7
model	0.8	1.4	12.0	18.2	67.6	14.8	14.5	22.4

Data source: Díaz-Giménez et al. (2011)

### 3 The impacts of fiscal stimulus programmes

Having set up and calibrated the model, we now turn to the main question of this article: what are the aggregate and heterogeneous effects on employment, consumption and output

<sup>7</sup>The steady state government spending to GDP ratio is calculated based on NIPA data from 1981Q1 to 2014Q2.

of intra- and inter-temporal transfers? To answer this question, we conduct two policy experiments. In the first experiment, there is a temporary uniform lump-sum transfer to all agents financed via public debt. This transfer is considered as intertemporal redistribution because public debt has to be repaid via future tax increase. In the second one, an intratemporal transfer takes place within the period from the wealthiest members to the least wealthy ones in the society.

### 3.1 Intertemporal Redistribution

#### 3.1.1 policy experiment set up

First, we consider a tax exempted uniform transfer,  $Tr_u$ , to all households, financed via public debt. The size of total transfer is set to be 1% of steady state government spending and for simplicity we assume that the transfer shock has zero persistence. Starting from the second period, public debt is repaid via income taxes. Following Bilbiie et al. (2013), we assume a general financing scheme that the proportionate part of the income tax rate  $a_3$  increases to repay public debt gradually:

$$\Delta a_3 = \phi_{td} \frac{B}{Y_{ss}}$$

where  $\Delta a_3$  is the change of tax rate,  $Y_{ss}$  is the steady state level of output;  $\phi_{td}$  is a parameter that gauges the speed of fiscal consolidation. To ensure debt sustainability, the response of taxes to public debt needs to obey:

$$\phi_{td} \in (r_{ss}, 1]$$

In our policy experiment, we use a modest rate of fiscal consolidation:  $\phi_{td} = 0.08$ .

### 3.1.2 effects of debt-financed uniform transfers

Fig. 1 displays the impulse response functions (IRFs) of  $a_3$ , labor supply, aggregate effective labor and output. Fig. 2 shows the IRFs of consumption and savings.<sup>8</sup> All IRFs except for  $a_3$  are percentage deviations from steady state. From these figures, we find that a debt-financed uniform transfer boosts aggregate consumption on impact while it causes a persistent contraction of output. Individual decisions are driven by four forces: transfers, income taxes, factor prices and the borrowing constraint, and each force has different importance to households differing in wealth and labor productivity.

***The role of distortionary taxation:*** A debt-financed transfer, in general, provides positive income effects but dampened by the increase in future taxes. Although the transfer is uniformly distributed among households, future tax burden is not equally distributed. In our policy experiment, public debt is repaid via an increase of the proportionate part of the income tax rate. As a result, wealthier households whose average income are higher<sup>9</sup> bear a higher increase in future tax burden. Hence, the overall income effect is a decreasing function in wealth. Consequently, labor supply on impact exhibits diverse responses: the bottom 20% of working-age households reduce the most labor supply while the top 20% of households even increase working hours. In the second period, the income tax rate jumps and gradually returns to steady state over time. As the income tax provides disincentive to work, the labor supply of middle and upper classes exhibit an inverse relationship with it.

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<sup>8</sup>The disaggregated impulse responses at each point of time are the group average responses of each quintile. One potential criticism is that, except for the impact period, the composition of households in each quintile may change, i.e. social mobility happens. However, as labor productivity is highly persistent, social mobility is slow. Moreover, our goal is not to track each individual's economic activities which are largely affected by idiosyncratic shocks, but to explore the consequences of aggregate shocks. Therefore, the group averaged responses satisfy our purpose.

<sup>9</sup>This is because wealthier households have more capital income as well as higher average labor productivity.

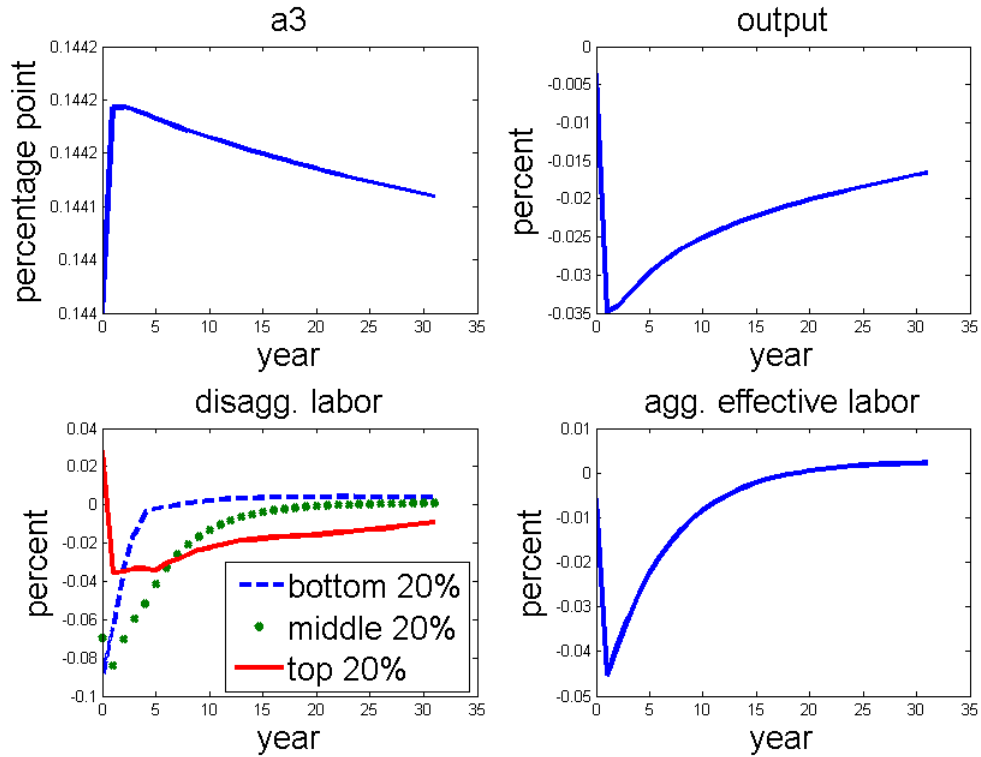


Figure 1: IRFs: tax rate, labor and output

***The role of the borrowing constraint:*** With the borrowing constraint and progressive taxation, labor supply of workers in the bottom 20% of population is determined in a more complicated way. Besides the disincentive to work, the rise of income tax rate also reduces labor income given any level of labor supply. Together with the persistent decline of the wage rate starting from the third period as shown in Fig. 3, less wealthy households are more likely to be financially constrained and have to supply more labor to resume consumption. Moreover, the lower wage rate drives down the marginal tax rate due to the progressive taxation, which encourages households to work more. Combining all these effects, labor supply of less wealthy workers gradually increase after the shock. On the other hand, as shown in the lower-right panel of Fig. 1, total effective labor supply is mainly driven by the income tax rate and shows a further drop when the tax rate hikes and gradually recovers to steady state.

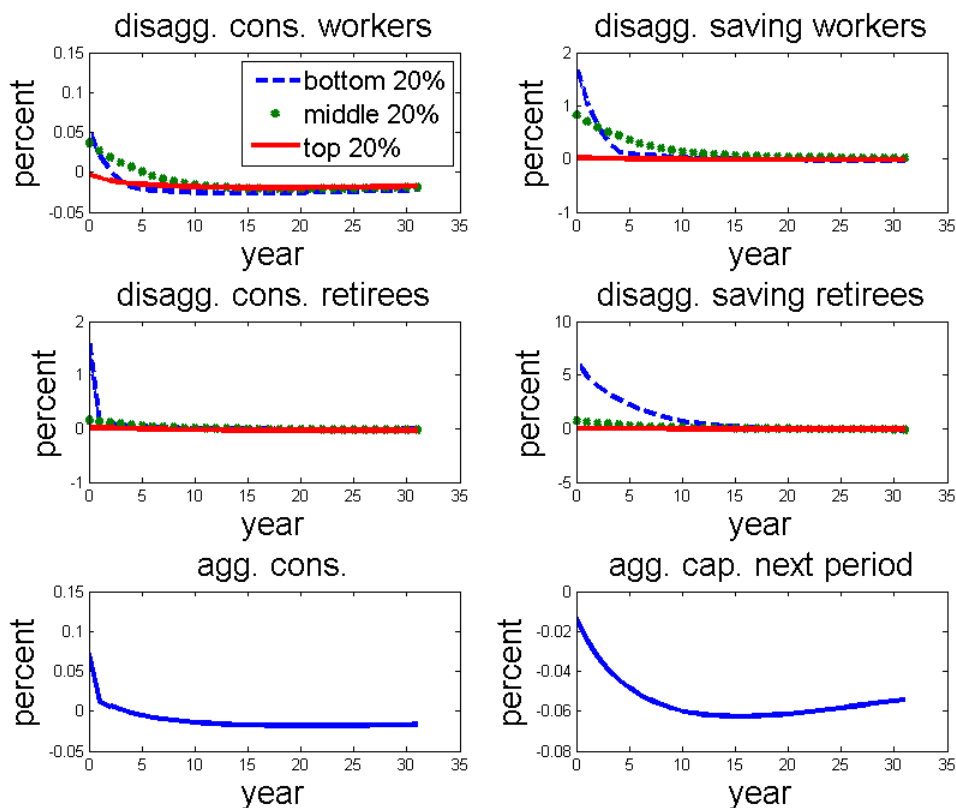


Figure 2: IRFs: consumption and savings

Fig. 2 displays impulse response functions of consumption and savings for working-age households and retirees. The responses on impact are also decreasing functions in wealth due to the asymmetric income effect. In the long run, consumption of middle and top classes working-age population return to steady state from above and below, respectively. Combining the effects of the borrowing constraint, the higher income tax rate and the lower wage rate, consumption of the least wealthy workers reverts fast and goes below steady state for a long period. For retirees, the borrowing constraint does not bind because they receive social security transfers each period, hence consumption of the least wealthy retirees does not fall below steady state.

On the other hand, since public savings decline and the wealthiest households lower their savings to smooth consumption, the aggregate capital stock encounters a persistent



contraction. As both capital stock and effective labor supply decline and recover slowly, total output exhibits a persistent contraction as shown in Fig. 1.

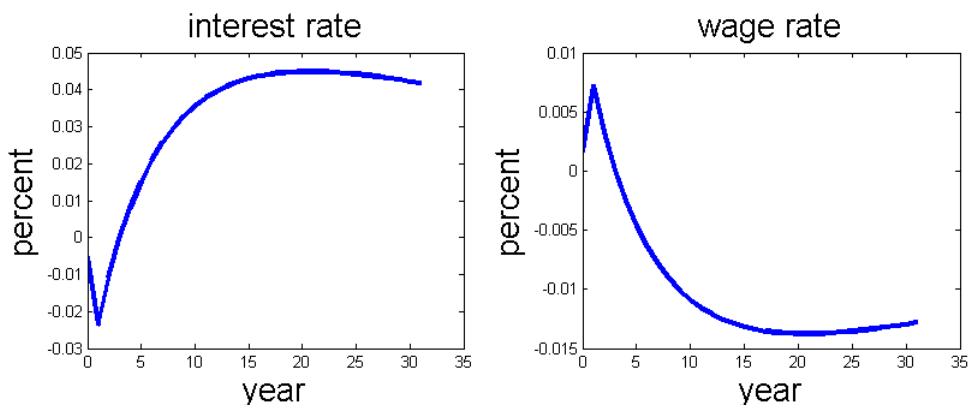


Figure 3: IRFs: factor prices

*General equilibrium effects:* Since the aggregate effective labor drops and further declines in the first two periods, marginal productivity of capital goes down while marginal productivity of labor rises. Hence, as shown in Fig. 3, the real interest rate drops and the real wage rate increases on impact and in the second period. In the long term, because both aggregate effective labor and capital stock suffer persistent decline, changes of factor prices depend on the relative speed of recovery. As the aggregate effective labor recovers faster, capital becomes relatively scarcer. Consequently, the real interest rate goes above the steady state while the real wage rate drops below in the medium and long run. The initial decline of the real interest rate further dampens the positive wealth effect of transfers especially to wealthy households, which enlarges the divergence of initial individual responses. Moreover, the initial lower real interest rate drives down the marginal income tax rate for any level of labor supply, which encourages households to work more. This effect is stronger to wealthy households as their capital income is high. In contrast, the higher real interest rate in the long run brings more capital income and drives up the marginal income tax rate, which discourages labor supply. These effects are also stronger to wealthier households. On the other hand, changes of the wage rate carry two opposite effects: it alters the return of each unit of effective labor supply but also pushes up the marginal income tax rate given any

level of labor supply. Hence, given wealth and labor productivity, labor supply is a nonlinear function of the wage rate.

**Consumption multipliers:** Following Bilbiie et al. (2013), the T-years present-value multipliers on aggregate consumption are defined as:

$$M_T^{agg} \equiv \frac{\partial(\sum_{i=0}^T \beta^i C_{t+i})}{\partial T r_u}$$

For  $T = 0$ , it is the impact multiplier.

Table 5 displays the present-value and impact aggregate consumption multipliers. In order to see the importance of the general equilibrium effects, we also calculate the consumption multipliers in an economy with constant interest rates and wage rates. That is, in this partial equilibrium economy the fiscal variables have the same paths as in the general equilibrium model, but factor prices are fixed at steady state levels. Households are fully informed about the partial equilibrium feature of the economy. The present-value aggregate consumption multiplier exhibits a nonlinear relationship with time. For each one dollar debt-financed transfer, the aggregate consumption rises by 17 cents on impact and its total increase is 21 cents in present value over the first three years. However, the present-value multiplier diminishes over time implying a contraction of aggregate consumption in the long run caused by the persistent contractions of effective labor and capital stock. Without general equilibrium effects, the present-value consumption multipliers are larger. The reasons are the following. If there was no factor price changes, households would receive more labor income but less capital income. Such differences could either encourage or discourage household consumption depending on the relative importance of income sources. Starting from the least wealthy households, as their main source of income is labor wage, their consumption would increase if there were no general equilibrium effects. For richer households, the labor income become less important, hence their consumption would increase by less without factor prices changes. In the aggregate level, since less wealthy households have

higher marginal propensity to consume, the aggregate consumption is higher in the partial equilibrium economy.

Table 5: aggregate consumption multipliers

	impact	3-years	10-years	15-years	20-years
general equilibrium	0.17	0.21	0.15	0.07	0.01
partial equilibrium	0.17	0.22	0.17	0.11	0.05

In order to see the heterogeneous impacts of transfers, we define the T-years present-value multipliers on consumption of each quintile of households as:

$$M_T^j \equiv \frac{\partial(\sum_{i=0}^T \beta^i \sum_{a \in A_{ji}} c_{t+i}(a))}{\partial T r_u}$$

where  $A_{ji}$  is the range of asset holding for the  $j$ th quintile at time  $i$ .

Table 6 shows the present-value consumption multipliers for each quintile of working-age households and retirees. Three findings need to be emphasized. First, the present-value multiplier goes up initially and diminishes over time for workers while it increases for retirees. Second, the multiplier decreases in wealth on impact and then become hump-shaped for workers; while it is always decreases in wealth for retirees. This result is due to the unequally distributed income effect and the borrowing constraint as discussed above. The effect of the borrowing constraint can be seen from the comparison of consumption multipliers between workers and retirees. As the constraint does not bind for the retirees because of social security transfers, the consumption multiplier does not exhibit a hump-shaped relationship with wealth in the medium and long run but instead decreases in wealth for the retired. Third, moving from the general equilibrium economy to the partial equilibrium one, the multiplier increases for working-age households while it decreases for the retired. That is because, under partial equilibrium, the relatively higher wage rate brings more income to workers, which would offset or even overcome the effect from relatively lower interest rates. For retirees, as they do not earn labor income and lose the benefit from higher interest rate in the general equilibrium economy, they have to reduce consumption.

Table 6: consumption multipliers

		quintile									
		1st		2nd		3rd		4th		5th	
		GE	PE	GE	PE	GE	PE	GE	PE	GE	PE
workers	impact	0.08		0.06		0.06		0.01		-0.02	
			0.08		0.06		0.05		0.01		-0.02
	3-years	0.11		0.12		0.12		0.01		-0.13	
			0.11		0.12		0.12		0.03		-0.11
	10-years	-0.02		0.08		0.11		-0.09		-0.56	
			0.01		0.09		0.14		0.00		-0.49
retirees	impact	0.85		0.18		0.13		0.07		0.04	
			0.85		0.17		0.13		0.07		0.04
	3-years	0.95		0.41		0.31		0.19		0.13	
			0.95		0.41		0.31		0.19		0.11
	10-years	1.04		0.67		0.54		0.38		0.12	
			1.03		0.62		0.49		0.34		-0.05

### 3.1.3 welfare implications

Following Krusell and Smith Jr (1999), we measure the welfare change in terms of percentage change in life time consumption, i.e. the Consumption Equivalent Variation (CEV, denoted as  $\lambda$ ). Given perfect foresight of the government transfer shocks, we can calculate the consumption equivalent variation along the balanced growth path which makes households indifferent between the government transfer shocks and the modified path. That is, we calculate  $\lambda$ , such that

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[ \frac{((1+\lambda)c_t)^{1-\sigma}}{1-\sigma} + \chi \frac{(\bar{l} - l_t)^{1-\theta}}{1-\theta} \right] = E_0 \sum_{t=0}^{\infty} \beta^t \left[ \frac{\tilde{c}_t^{1-\sigma}}{1-\sigma} + \chi \frac{(\bar{l} - \tilde{l}_t)^{1-\theta}}{1-\theta} \right]$$

where  $c_t$  is consumption in the economy with government transfer shocks, while  $\tilde{c}_t$  is that in the economy without government transfer shocks. Please note that a positive  $\lambda$  represents a welfare loss and a negative one represents an improvement in welfare.

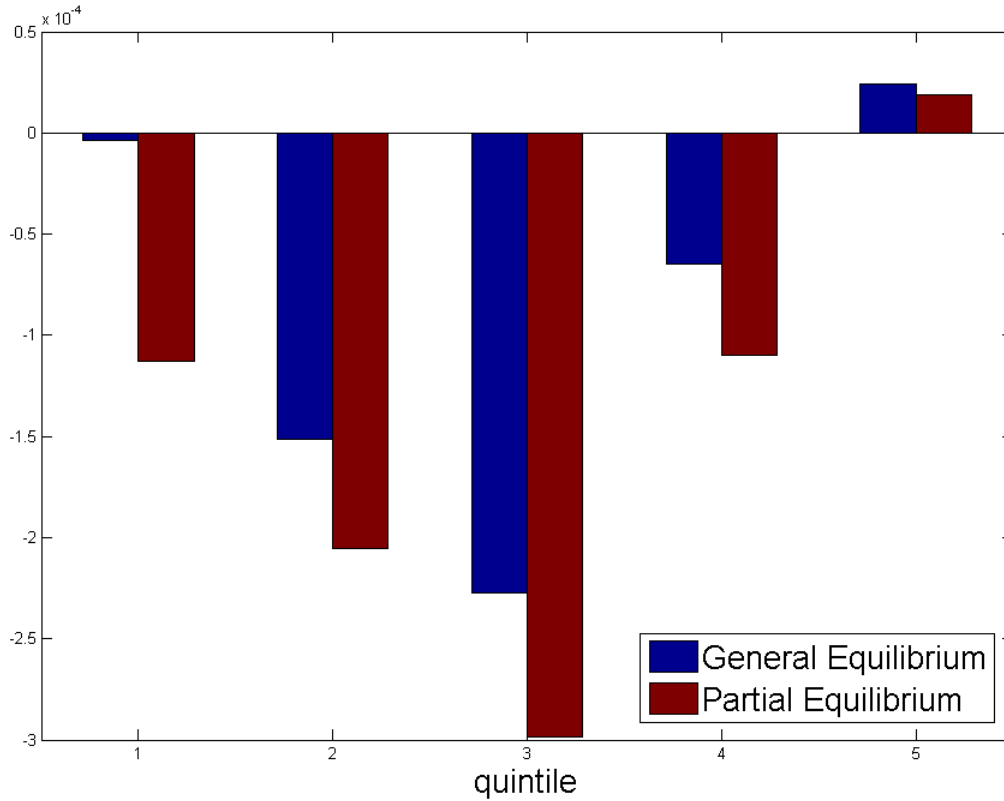


Figure 4: Consumption equivalent variations: workers

Fig. 4 shows the consumption equivalent variations for each quintile of working-age households in both general and partial equilibrium economies. The left (blue) bar is the  $\lambda$  in the benchmark economy, while the right (red) bar is that with constant levels of factor prices. Several findings need to be emphasized. First, the debt-financed uniform transfer improves welfare for the first four quintiles but reduce welfare for the wealthiest group. More specifically, the consumption equivalent variation exhibits an inverse hump-shaped relationship with wealth, i.e. workers with a medium level of wealth gain the most in welfare. The unequally distributed income effect improves welfare more for less wealthy households. However, the borrowing constraint asks low income households to work more and consume less during high tax rate and low wage rate periods. Hence, workers who are more closer to the borrowing constraint get less welfare gain than those who are far from

the constraint. The inverse hump-shaped consumption equivalent variation is, therefore, formed by the combined effects of transfers, tax changes and the borrowing constraint. This result also indicates that the debt-financed uniform transfer reduces inequality in terms of welfare between the richest households and others but enlarges inequality within the mid- and low-income classes. In addition, without factor price changes, all workers would have further welfare gain. This is because the relatively higher wage rate in partial equilibrium benefits all workers.

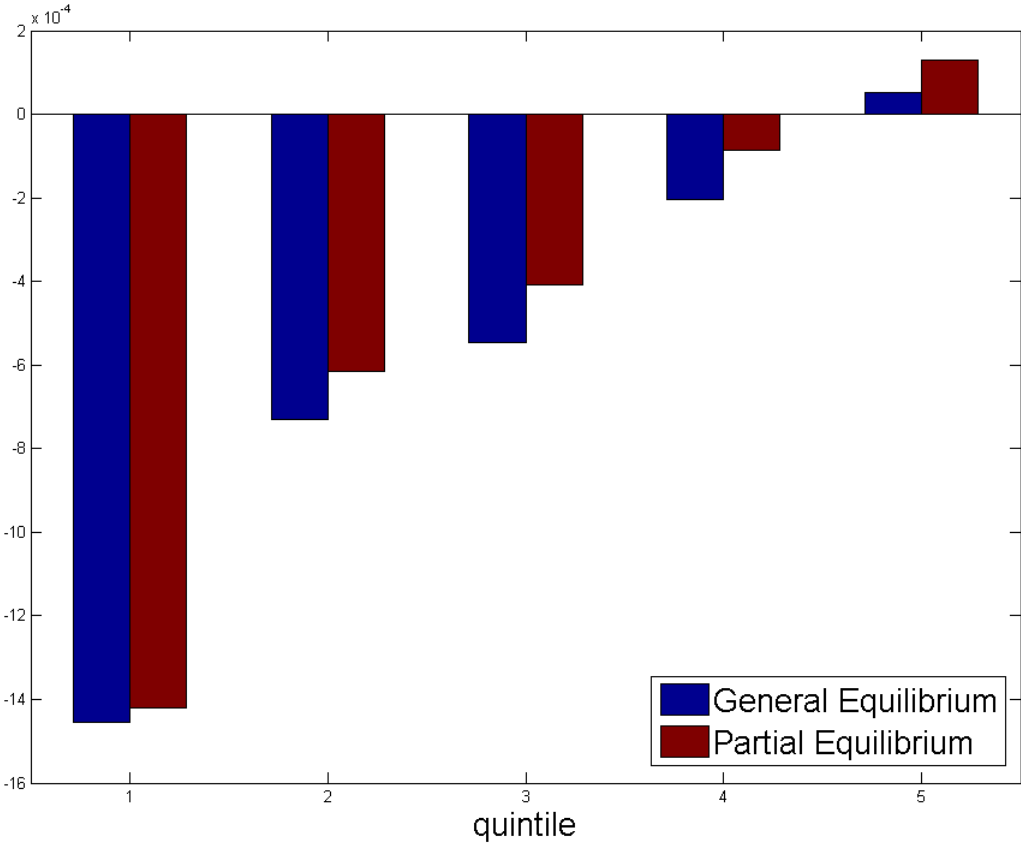


Figure 5: Consumption equivalent variation: retirees

Fig. 5 displays the consumption equivalent variations for each quintile of retirees. The welfare gain decreases in wealth reflecting the unequally distributed income effects of the debt-financed transfer. The borrowing constraint will not bind for retired households as they receive social security transfers every period. If we fix factor prices, the welfare gain

shrinks for all retirees. The general equilibrium effects affect retired households only through changes of the real interest rate. In the partial equilibrium economy, there is no persistent increase in the real interest rate, so households lose the benefits from capital income increases.

## 3.2 Intratemporal Redistribution

### 3.2.1 policy experiment set up

In our second policy experiment, we engineer a one time transfer from the wealthiest households of the economy to the least wealthy within a period. We call this type of transfer a Robin-Hood intratemporal transfer. Specifically, we tax the top 20% of households and make transfers to the bottom 20% of households. We assume that the additional tax is deducted from after-tax income and the transfer is not taxable. As Oh and Reis (2012) pointed out, there is no study on how transfers are distributed across different groups in the population. We proceed by considering a systematic policy rule that satisfies two principles. First, households who hold less asset receive more and those who have the most asset pay the most, so  $Tr(\cdot)$  is decreasing in  $a$  and  $T(\cdot)$  is increasing in  $a$ . Second, households would not change their positions in the population distribution as a result of receiving or paying transfers/taxes. For households who receive transfers, the amount of transfer is given by:

$$Tr(a) = \gamma_k \left(1 - \frac{a}{\bar{a}}\right)^{\theta_k} I(a \leq \bar{a})$$

where  $I(a \leq \bar{a})$  is an indicator function, that is households can receive transfers only if their wealth is lower than  $\bar{a}$ .  $\gamma_k > 0$  and  $0 < \theta_k \leq 1$  are parameters that determine the size and curvature of transfers.  $\gamma_k$  is greater than zero because transfers have to be positive. We want  $Tr(a)' < 0$  such that less wealthy households receive more, so  $\theta_k > 0$ . Moreover, as the wealth gap between households is an increasing function in  $a$ , we want  $Tr(a)'' \leq 0$ . Hence  $0 < \theta_k \leq 1$ .

For households who have to pay lump-sum tax to fund the transfer, the amount of tax

is given by:

$$T(a) = \gamma_w \left( \frac{a - \underline{a}}{a_{max} - \underline{a}} \right)^{\theta_w} I(a \geq \underline{a})$$

where  $I(a \geq \underline{a})$  is an indicator function, that is households pay lump-sum taxes only if their wealth is greater than  $\underline{a}$ .  $\gamma_w > 0$  and  $\theta_w \geq 1$  are parameters that determine the size and curvature of taxes. Similar to the rule for transfers, the additional tax is positive and its increment is getting larger and larger, i.e.  $T(a)' > 0$  and  $T(a)'' \geq 0$ .  $a_{max}$  represents the highest level of asset holding by households in stationary equilibrium. In our experiment, total amount of taxes/transfers is set to be 1% of steady state government spending and  $\theta_k = \theta_w = 1$ .

### 3.2.2 effects of intratemporal transfers

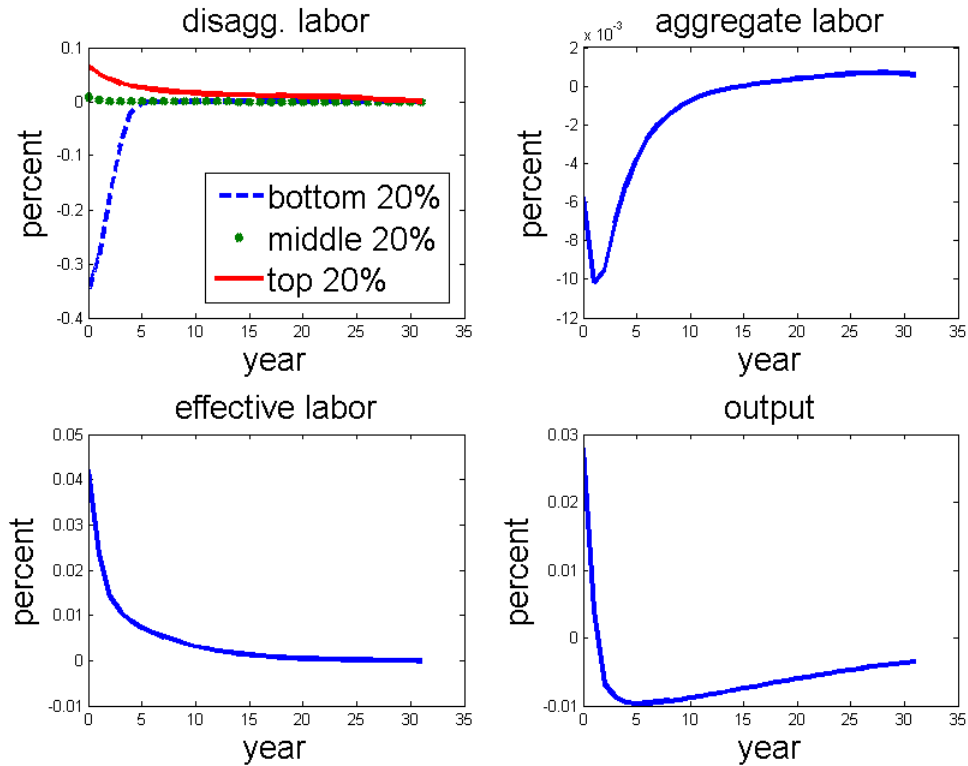


Figure 6: IRFs: labor, effective labor, and output



Fig. 6 displays the impulse response functions (IRFs) of labor supply, effective labor and output. With imperfect insurance, transfers from the wealthiest households to those with low wealth boost effective labor and output through the income effect channel. To be more specifically, the upper-left panel of Fig. 6 plots labor supply of different groups of working-age households. The bottom 20% of households decrease labor supply as their wealth has increased; while the negative wealth effect asks the top 20% of households to work more. For those who do not receive transfers nor pay taxes, their labor supply is barely changed. In addition, the change of labor supply of less wealthy households exhibits a larger size because the amount of transfers is relatively larger compared to their wealth. As a result, aggregate labor supply decreases. However, since the wealthiest workers are on average more productive, aggregate effective labor increases as shown in the lower-left panel of Fig. 6. Output, therefore, exhibits an expansion on impact.

Fig. 7 displays the IRFs of consumption and savings. The wealth effect boosts consumption of the bottom 20% households; while it slightly dampens consumption of the wealthiest group. The size of the consumption response of the payees is relatively small, because those households are willing and have the capability to smooth consumption via labor and savings. In other words, the recipients of transfers have on average a higher marginal propensity to consume (MPC) than the payees, hence aggregate consumption is boosted on impact.

For savings, although less wealthy households increase their savings and rich households only barely change their savings, the aggregate savings still decline. That is because asset is highly concentrated: top 20% of households hold more than 83% of total wealth and bottom 20% of households almost have no wealth. The persistent decline of savings, as shown in the lower-right panel of Fig. 7, causes long-term consequences. Starting from the second period, the top 20% group gradually reduces labor supply to steady state and the bottom 20% of households increase labor supply. Aggregate labor gradually recovers from below while aggregate effective labor returns to steady state from above. Together with a slowly recovery of capital stock, output declines below trend after the impact of the shock and goes

back to steady state in a low speed.

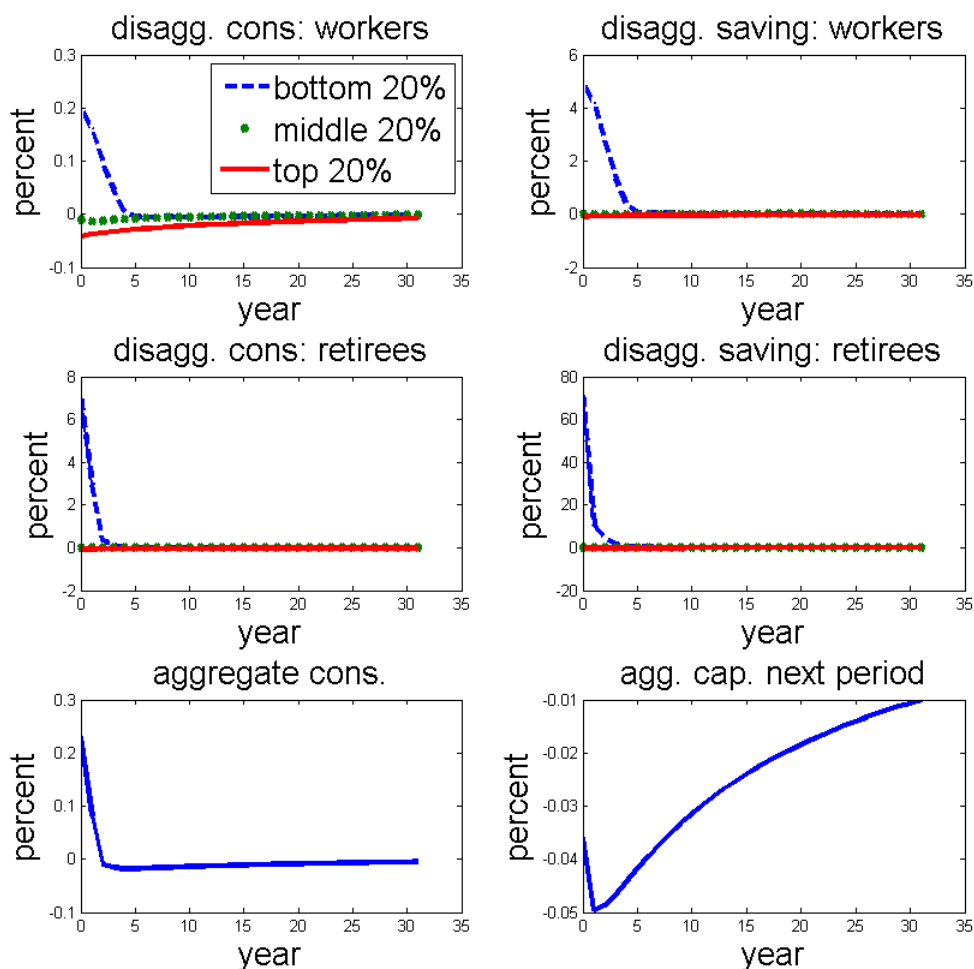


Figure 7: IRFs: consumption and savings

There is another channel that transfers could affect economic activities: the general equilibrium channel. As the interest rate and wage rate are determined by marginal productivity of capital and labor, respectively, the responses of aggregate effective labor and capital will lead to changes in factor prices. Those changes in factor prices lead to heterogeneous impacts on households. As most assets are held by wealthy households, changes in the interest rate have larger impacts to them. In this case, the interest rate jumps as a result of the increase in effective labor supply, which will dampen the negative wealth effects to the payees pre-

venting them from further increasing their labor supply. Changes in the wage rate generate substitution effects. In this case, the wage rate declines, which discourages labor supply for all households. Hence, without the general equilibrium effects, the aggregate effective labor and the output would jump higher on impact together with a less severe output contraction in the long-run.

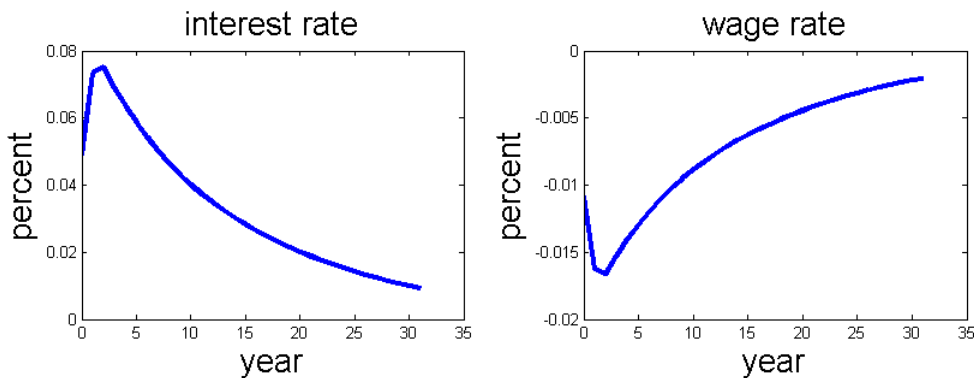


Figure 8: IRFs: interest rate and wage rate

### 3.2.3 fiscal multipliers:

Table 7 displays the present-value aggregate consumption and output multipliers. In our experiment, the payees will decrease consumption while the recipients will consume more in response to this transfer shock. Since the recipients have higher MPC, the aggregate consumption is boosted. For each one dollar Robin-Hood intratemporal transfer, the aggregate consumption rises by 55 cents on impact and its total increase is 72 cents in present value over the first three years. Output multiplier, on the other hand, is 0.12 on impact but turns into negative values in the long run. That is because the wealthiest households cut savings due to the negative income effect, which leads to a decline in aggregate capital stock<sup>10</sup>.

<sup>10</sup>The recipients increase savings, but their total asset is negligible.

Table 7: fiscal multipliers

consumption multipliers				
impact	3-years	10-years	15-years	20-years
0.55	0.72	0.54	0.48	0.44
output multipliers				
impact	3-years	10-years	15-years	20-years
0.12	0.11	-0.07	-0.14	-0.18

## 4 Conclusion

This paper investigates the short-run and long-run aggregate impacts, heterogeneous effects and welfare implications of intra- and inter-temporal redistribution based on a model with idiosyncratic labor productivity shocks, financial imperfections, distortionary taxation and public debt. We find that, under flexible prices, a uniform debt-financed transfer has a positive impact on consumption on impact but not in the long-run; and it causes a persistent contraction of output. Moreover, due to distortionary taxation and the borrowing constraint, the uniform transfer leads to heterogeneous impacts and welfare implications on households. In addition, it reduces inequality in terms of welfare between the richest households and others but enlarges inequality within the mid- and low-income classes. On the other hand, a Robin-Hood intratemporal redistribution to low wealthy households is found to be expansionary on private consumption and effective labor hours. In addition, its output multiplier is positive on impact but turns into a negative value in the long run. Finally, general equilibrium effects are found to have a significant role in shaping individual decisions.

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