# Government Sector and the Government Spending Multipliers

## Rong Li\*

## April 2014

Distinguishing two components of government spending is important, because the shocks of these two components have opposing effects on the private sector. Quantitatively, a one dollar increase in government purchases on goods and services from the private sector leads to a \$1.99 to \$2.52 increase in the private sector output. However, a one dollar increase in government purchases on goods and services from the government sector results in a \$1.64 to \$1.81 loss of the private sector output. The policy implication is that in order to stimulate the private sector output, the government needs to spend money in the private sector instead of the government sector. Moreover, this paper can potentially reconcile the conflicts among different identification strategies regarding the response of private consumption. That is, the response of consumption varies with the structure of the identified government spending shocks.

<sup>\*</sup>li.1086@osu.edu, Department of Economics, The Ohio State University. I would like to thank Bill Dupor, Paul Evans and Pok-Sang Lam for advice. I am also grateful to the useful feedback from Jinhui Bai and a number of conference audiences.

# 1 Introduction

This paper studies the effects of shocks to different components in government spending. Many researchers assume that the goods and services purchased by the government and households are homogenous when estimating the effects of government spending shocks. For example, Blanchard and Perotti (2002), Mountford and Uhlig (2009) and Ramey (2011) identify government spending shocks using different approaches but they all maintain the above assumption. However, there are two main components in total government expenditure: the purchase of goods and services from the private sector, and the purchases of goods and services from the government sector. The latter one is the compensation to general government employees plus the general government consumption of fixed capital. In the National Income and Product Accounts (NIPA) table 1.3.5, this component is classified as the general government sector value added. Moreover, from 1947Q1 to 2012Q4, the average government spending share of purchases from the private sector is 0.40 and the average share of the other component is 0.60. So government expenditure in the government sector is a significant component in government spending. However the households do not directly pay for the goods and services from the government sector, implying that the consumption bundles of the government and households are different.

Therefore, without distinguishing the two components of government spending, the study of the effects of government spending shocks may lead to an inaccurate conclusion. Intuitively, an increase of spending in any component results in negative wealth effects due to the expansion of government's usage of economy resources. This negative wealth effect will force the households to work more and consume less. However, an increase in purchases of goods from the private sector will expand the hours worked in the private sector while an increase in the demand for government sector goods leads to an increase in government sector hours. More importantly, the latter one will reallocate the labor from the private sector to the government sector due to the decrease in households consumption and hence the decrease in the demand for private sector output.<sup>1</sup>

To get more precise analysis of the effects of government spending shocks, I distinguish these two components. I first construct an analytical two-sector New Keynesian model. The model illustrates that an increase in government purchases on private output has a positive impact on the private output, while an increase in government purchases of government output lowers the private output.

I then study the quantitative effects of shocks to the two components. First, I identify the shocks to the two components of government spending using an extended version of the narrative approach and defense news series as in Ramey (2011). Second, I construct a standard New Keynesian model with habit formation, sticky prices, capital adjustment costs, and most importantly a distinction between government purchases of private and government output; and estimate the model's parameters by

 $<sup>^1\</sup>mathrm{Note}$  that the government demand of private output does not change.

matching the impulse responses of the model and the VAR. Third, I conduct counterfactual experiments by varying one type of government spending while keeping the other constant to illustrate how the isolated shocks affects the economy. I find that a one dollar increase in government purchases on goods and services from the private sector leads to a \$1.99 to \$2.52 increase in the private sector output and a \$1.23 to \$1.44 increase in private consumption. However, a one dollar increase in government purchases on goods and services from the government sector results in a \$1.64 to \$1.81 loss of private sector output and lowers private consumption by \$0.23 to \$0.25.

Based on the above findings, this paper can potentially reconcile the conflicts regarding consumption response to a government spending shock. That is, the response of consumption varies with the structure of the identified government spending shock. If the identification method identifies a government spending shock with more money spent in the private sector, it tends to provide a positive impact on consumption, and vice versa. I find that the Structural Vector autoregression (SVAR) approach identifies a government spending shock with more money spent in the private sector, while the narrative approaches identify government spending shocks with more expenditure in the government sector. Hence the consumption response is positive with the SVAR approach; it is around zero with the Defense News approach; and it is negative with the War Dates approach.

The paper is organized as follows. Section 2 presents the empirical findings. Section 3 study the effects of different components in government spending analytically. Section 4 uses a New Keynesian model with the capability of matching the impulse responses from the data to study the quantitative effects. Section 5 provides empirical evidence on reconciling the conflicts regarding the consumption response to a government spending shocks. Section 6 concludes.

# 2 Government Spending Multipliers in An analytical Model

In this section, I first discuss in detail how to distinguish between the two components of government spending. Then, I construct a simple analytical model to illustrate how different components of government spending affects the private sector output.

### 2.1 The two components in government spending

Traditionally, researchers assume that the goods and services purchased by the government and households are homogenous. However, there are two components in government spending. In the National Income and Product Accounts (NIPA), government purchases include both purchases of goods and services from the private sector and from the government sector. Since households do not directly pay for the goods and services provided by the government sector, the consumption bundles of government and households are different. Figure 1 illustrates the fact. The top panel divides GDP by producer.  $Y_2$  represents the output by the government. Examples include defense services, education services, water and other sanitation services. In the NIPA 1.3.5,  $Y_2$  is the general government sector value added which equals to compensation to general government employees plus the general government consumption of fixed capital.  $Y_1 = GDP - Y_2$  is the output of private sector. The middle panel divides GDP by purchaser, G is the NIPA category of government purchases of goods and services; the rest of GDP is purchased by households, used as consumption, investments or net exports. The bottom panel combines these two categorizations. As it shows, government purchases G include both the goods and services provided by private sector  $(G_1)$  and government sector  $(G_2)$ , while households purchases do not include the goods and services from the government sector. Therefore, the goods and services purchased by the government and households are not homogenous.

Finn (1998) used a dynamic neoclassical model to show that increases in total government spending resulting from an increase in government employment and resulting from an increase in purchases of goods from the private sector have opposite effects on private sector output, employment, and investment. Cavallo (2005) showed that allowing for the distinction between the two main components of government consumption improves the quantitative performance of the neoclassical growth model. Therefore, the analysis of government spending shocks without distinguishing the two sectors goods may lead to an inaccurate conclusion. Intuitively, an increase in  $G_1$  or  $G_2$  both have negative wealth effects because the expansion of government's usage of economy resources. This negative wealth effect will encourage the households to work more, and therefore total hours worked rise. However, an increase in  $G_1$ will only expand the hours worked in the private sector while an increase in  $G_2$  will generally lead to an increase in government sector hours. Moreover, more  $G_2$  may also result in a sectoral employment reallocation which moves the labor time from the private sector to the government sector. Different from Finn (1998) and Cavallo (2005), I investigate the effects of shocks to different components in government spending in a New Keynesian economy and provide quantitative analysis.

Figure 1: distinction Division of Goods and Services by Producer

Government Sector Output	Private sector Output	
Y2	Y1	

Division of Goods and Services by Purchaser

Government Purchases	Households Purchases
G	C+I

**Combined Categorization** 

G2=Y2 G1	Y-G=Y1-G1
----------	-----------

## 2.2 An analytical New Keynesian model

There are two production sectors in the model: a private sector, denoted as Sector 1, and a government sector, denoted as Sector 2. Households consume goods from the private sector while the government purchases goods from both sectors. Moreover, Sector 1 is a monopolistic competitive sector with Calvo type of sticky prices. Sector 2 sells its output to the government at cost. The production functions are  $Y_{it} = N_{it}$ , for i = 1, 2, where  $N_{it}$  is the labor in sector *i*. The total labor input is  $N_t = N_{1t} + N_{2t}$ , and the steady state ratio of  $\frac{N_i}{N}$  is equal to  $\frac{Y_i}{Y}$ .

Households.—The economy is populated by a representative household, whose lifetime utility, U, is given by

$$U = \sum_{t=0}^{\infty} \beta^{t} E_{0} \left[ \frac{C_{t}^{1-\sigma}}{1-\sigma} - \Psi_{N} \frac{N_{t}^{1+\theta}}{1+\theta} \right]$$
(1)

Here  $E_0$  is the conditional expectation operator, and  $C_t$  and  $N_t$  denote consumption and hours worked at time t, respectively. We assume  $\sigma, \theta > 0$  and households do not value public consumption.

The household budget constraint is given by

$$P_t C_t + \frac{B_t}{R_t} = W_t N_t + B_{t-1} + D_t + T_t$$
(2)

where  $D_t$  denotes firms' profits and  $T_t$  denotes lump-sum taxes paid to the government.  $B_t$  denotes the quantity of one-period bonds purchased by the household at time t.  $P_t$  denotes the price level and  $W_t$  denotes the nominal wage rate. Finally, the variable  $R_t$  denotes the gross nominal interest rate in period t. The household's problem is to maximize the lifetime utility given by (1) subject to the budget constraint (2) and the condition

$$E_0 lim_{t \to \infty} B_t / [R_0 R_1 \cdots R_t] \ge 0 \tag{3}$$

Firms.–The final good is produced by competitive firms using the technology

$$Y_{1t} = \left[\int_0^1 y_{1t}(i)^{\frac{\epsilon-1}{\epsilon}} di\right]^{\frac{\epsilon}{\epsilon-1}}, \epsilon > 1$$
(4)

where  $y_{1t}(i), i \in [0, 1]$ , denotes intermediate good *i*.  $\epsilon > 1$  is the demand elasticity of differentiated goods.

Profit maximization implies the following demand function:

$$y_{1t}(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\epsilon} Y_{1t}$$
(5)

where  $P_t(i)$  denotes the price of intermediate good *i* and  $P_t$  is the price of the homogeneous final good.

The intermediate good  $y_{1t}(i)$ , is produced by a monopolist in the private sector using the following production function:

$$y_{1t}(i) = N_{1t}(i)$$
 (6)

where  $N_{1t}(i)$  denotes the ith monopolist's employment. There is no entry or exit into the production and the monopolist is subject to the Calvo-type price-setting. It can optimize its price,  $P_t(i)$ , with probability 1 - v, otherwise, the firm sets

$$P_t(i) = P_{t-1}(i)$$

The discounted profits of the ith intermediate production firm are given by

$$E_t \sum_{j=0}^{\infty} \beta^{t+j} \lambda_{t+j} [P_{t+j}(i)y_{1t+j}(i) - (1-v)W_{t+j}N_{1t+j}(i)]$$
(7)

as in Christiano et al. (2011),  $v = \frac{1}{\epsilon}$  denotes an employment subsidy that corrects the inefficiency created by the presence of monopoly power, in steady state.  $\lambda_{t+j}$  is the multiplier on the household budget constraint.

Firm i maximizes (7) subject to the Calvo-type price-setting friction, the demand function (5), and the production function (6).

Monetary policy.—The monetary authority uses the nominal interest rate,  $R_t$  as the instrument to conduct monetary policies. It determines  $R_t$  following a Taylor rule:

$$R_t = R_{t-1}^{\rho_r} \pi_t^{\phi_1(1-\rho_r)} (Y_{1t} + Y_{2t})^{\phi_2(1-\rho_r)}$$
(8)

where  $\rho_r \in (0, 1)$ . The monetary authority sets the nominal interest rate based on the last period's rate, inflation  $\pi_t$ , output of private sector  $Y_{1t}$ , and output of government sector  $Y_{2t}$ .

Fiscal policy.-Government spending evolves according to:

$$G_{it+1} = G_{it}^{\rho_i} exp(\varepsilon_{it+1})$$

for i = 1, 2.

 $\varepsilon_{it}$  are i.i.d. shocks with zero mean. For simplicity, we assume that government spending and the employment compensation are financed with lump-sum taxes. Because Ricardian equivalence holds in this economy, the exact timing of these taxes is irrelevant.

The resource constraints are:

$$C_t + G_{1t} = Y_{1t}$$

and

$$G_{2t} = Y_{2t} = N_{2t}$$

The labor market clearing condition is

$$N_t = N_{1t} + N_{2t}$$

and  $N_{1t} = \int_0^1 N_{1t}(i) di$ .

Here  $G_{it}$  is the level of government spending in sector i.

A "monetary equilibrium" is a collection of stochastic processes

$$\{C_t, N_t, N_{1t}, N_{2t}, W_t, P_t, Y_{1t}, R_t, P_t(i), y_{1t}(i), N_{1t}(i), v_t, B_t, \pi_t\}$$

such that for given  $\{G_{1t}, G_{2t}\}$  the household and firm problems are satisfied, the monetary and fiscal policy rules are satisfied, markets clear, and the resource constraints are satisfied.

The equilibrium can be described by the following log-linearized equations

$$\frac{1}{1-g_1}\hat{y}_{1t} - \frac{g_1}{1-g_1}\hat{g}_{1t} = \frac{1}{1-g_1}E_t\hat{y}_{1t+1} - \frac{g_1}{1-g_1}E_t\hat{g}_{1t+1} - \frac{1}{\sigma}E_t[\beta(R_{t+1}-r) - \pi_{t+1}] \quad (9)$$

$$\pi_t = \beta E_t \pi_{t+1} + \kappa \left[ \sigma \left( \frac{1}{1 - g_1} \hat{y}_{1t} - \frac{g_1}{1 - g_1} \hat{g}_{1t} \right) + \theta \left( \frac{Y_1}{Y} \hat{y}_{1t} + \frac{Y_2}{Y} \hat{g}_{2t} \right) \right]$$
(10)

$$R_{t+1} = r + \rho_r (R_t - r) + \frac{1 - \rho_r}{\beta} (\phi_1 \pi_t + \phi_2 (\frac{Y_1}{Y} \hat{y}_{1t} + \frac{Y_2}{Y} \hat{g}_{2t}))$$
(11)

where  $g_{it}$  is the government purchases in sector i and  $g_1 = \frac{G_1}{Y_1}$ .  $\kappa \equiv \frac{(1-v)(1-\beta v)}{v}$ . The first equation is the households intertemporal Euler equation. The second equation is the inflation Euler equation, and the third one is the monetary policy. A variable  $\hat{z}_t$  denotes the percentage deviation of  $Z_t$  from its nonstochastic steady-state value.

### 2.3 Effects of government spending shocks

I use the above model to study the effects of shocks in different components of government spending. Conjecturing a solution of the form:

$$\pi_t = \theta_1 \hat{g}_{1t} + \theta_2 \hat{g}_{2t}$$
$$\hat{y}_{1t} = \alpha_1 \hat{g}_{1t} + \alpha_2 \hat{g}_{2t}$$

There is a unique solution of this form, in which

$$\alpha_1 = \frac{(1-\rho_1) + (\phi_1 - \rho_1)\frac{\kappa}{1-\beta\rho_1}}{(1-\rho_1) + \frac{1}{\sigma}\phi_2\frac{Y_1}{Y}(1-g_1) + (\phi_1 - \rho_1)\frac{\kappa}{1-\beta\rho_1}(1 + \frac{1-g_1}{\sigma}\theta\frac{Y_1}{Y})}g_1$$

$$\alpha_2 = -\frac{\phi_2 + (\phi_1 - \rho_2)\frac{\kappa}{1 - \beta \rho_2}\theta}{(1 - \rho_2) + \frac{1}{\sigma}\phi_2\frac{Y_1}{Y}(1 - g_1) + (\phi_1 - \rho_2)\frac{\kappa}{1 - \beta \rho_2}(1 + \frac{1 - g_1}{\sigma}\theta\frac{Y_1}{Y})\frac{1}{\sigma}\frac{Y_2}{Y}(1 - g_1)}$$

Under the calibration that ensures a stationary equilibrium<sup>2</sup>,  $\alpha_1 > 0$  and  $\alpha_2 < 0$ . Therefore, shocks in  $G_{1t}$  have positive effect on the private sector output while

<sup>&</sup>lt;sup>2</sup>A stationary equilibrium requires that  $\rho_1 < 1$ ,  $\rho_2 < 1$ ,  $\phi_1 > 1$ , and  $\phi_2 > 0$ .

shocks in  $G_{2t}$  have negative effect on the private sector output. Intuitively, suppose there is a positive increase in  $G_{1t}$  and an unchanged level of  $G_{2t}$ . Then, there is a negative wealth effect because government increase the usage of private resource. In response to this negative wealth effect, households increase  $N_t$ . Because  $G_{2t}$  is fixed,  $N_{2t}$  is constant. Therefore, the increase in  $N_t$  occurs through the increase in  $N_{1t}$ . Consequently,  $Y_{1t}$  increases. On the other hand, consider a positive increase in  $G_{2t}$ and a constant level of  $G_{1t}$ . There is a negative wealth effect as usual. However, the increase in total labor supply  $N_t$  occurs through the increase in  $N_{2t}$  and the decrease in  $N_{1t}$ . The reason is that the negative wealth effect asks the households to increase labor supply  $N_t$  but to reduce consumption  $C_t$ . And because  $G_{1t}$  is fixed, the overall demand of private sector output  $Y_{1t} = C_t + G_{1t}$  falls. Alternatively, from Eq. 10, an increase in  $G_{2t}$  pushes up the real marginal cost and inflation. As a result, the nominal interest rate will be higher, then, from Eq. 9, households reduce consumption. Thus  $Y_{1t}$  falls. Therefore, the effect of an increase in  $G_{2t}$  is negative on private sector output. <sup>3</sup>

I then study the quantitative effects of shocks to the two components on the private sector by the following way. First, I identify the shocks to the two components of government spending and get the impulse responses using an extended version of the narrative approach and defense news series in Ramey (2011). Second, I construct a standard New Keynesian model with a distinction between government purchases of private and government output; and estimate the model's parameters by matching the impulse responses of the model and the VAR. Third, I conduct counter-factual experiments by varying one type of government spending while keeping the other constant to illustrate how the isolated shocks affect the economy. I also calculate the government spending multipliers on the private output and consumption for each component.

# 3 Identifying Government Spending Shocks

This section identifies the unexpected changes in government spending. I use the defense news series in Ramey (2011) to identify the government spending shocks. I find for each one dollar of total government spending increase, 44 cents are used as the compensation of general government employees or government consumption of fixed capital and 46 cents are spent in purchasing goods or services from the private sector.<sup>4</sup> In addition, I get the impulse responses of GDP, private output, investment inflation, and nominal interest rate which are used as the targets in the estimation.

<sup>&</sup>lt;sup>3</sup>In the appendix, I discuss the effects on wages and inflation.

<sup>&</sup>lt;sup>4</sup>10 cents are missing due to the use of average ratio of  $G_i$  to G. During the period of 1947-2012, these ratios change appreciably.

#### 3.1 An extended version of the narrative approach

In this subsection, I use the defense news series to identify government spending shocks and explore how private sector output responds to a defense news shock. Blanchard and Perotti (2002) identify government spending shocks using a Choleski decomposition in which the government spending is ordered first. Ramey (2011) argues that the standard SVAR misses the timing of the shocks, so she uses a narrative approach to identify the government spending shocks to avoid the potential anticipation effects. Following Ramey (2011), I use the defense news variables from Ramey (2011) to identify government spending shocks in the VAR.<sup>5</sup> This is an extended version of the narrative approach, because it measures the government spending changes due to exogenous foreign political events and uses more information. Another advantage of this approach is that it can provide the responses of each component in government spending to a shock to the defense news, while other methods cannot. The defense news series was constructed by reading periodicals in order to measure the public's expectations.<sup>6</sup> In this approach, the defense news variable is ordered before other variables in the VAR.

The basic empirical specification is:

$$A(L)Z_t = C + D_1 t + D_2 t^2 + U_t$$
(12)

where  $Z_t$  is a 6 × 1 vector of variables. The first 5 variables in  $Z_t$  are fixed including the defense news variable, the log of real per capita government spending, the log of real per capita GDP, the 3 month T-bill rate, and the Barro and Redlick (2011) average marginal income tax rate and the 6th in  $Z_t$  is a variable of interest. A(L) = $A_0 + A_1L + ... + A_4L^4$ , where L is the lag operator. The 3 month T-bill rate and the average marginal income tax rate are used to control for monetary and tax policy.<sup>7</sup>

To the fixed five variables, I rotate a series of the sixth variables. This is a widely used strategy in the literature to avoid using too many variables in the VAR.<sup>8</sup> The rotated variables are  $ln(G_1)$  government spending on private output, and  $ln(G_2)$  government spending on the government sector output.  $G_2$  is taken from NIPA table 1.3.5 which is the gross value added by the general government including compensation of general government employees plus the general government consumption of fixed capital. And  $G_1 = G - G_2$ , where G is the total government expenditure taken from NIPA table 1.1.5. That is, the government spending on private output is the difference between total government spending and its spending on the government sector. The average  $G_1$  share of total government spending is 0.40 and the average  $G_2$  share of total government spending is 0.60 in the period of 1947Q1-2012Q4.

 $<sup>{}^{5}</sup>I$  use quarterly data from 1947 to 2008 for the defense news approach.

 $<sup>^{6}</sup>$ See Ramey (2011) for details.

<sup>&</sup>lt;sup>7</sup>Rossi and Zubairy (2011) emphasize analysis of fiscal policy should always control for monetary policy and vice versa.

<sup>&</sup>lt;sup>8</sup>See Burnside et al. (2004).

Figure 2 shows the responses to a defense news shock. The responses are normalized such that the total government spending's peak response is equal to one. After a defense news shock, total government spending and each of its components rise, peaking six quarters after the shock and returning to their normal level after 20 quarters. The peak response of  $G_1$  is larger than the peak response of  $G_2$  as well as G, meaning that the government spends more money in the private sector than it normally does. However, it can be seen from Table 1 that there is still a large amount of money that goes to the government sector. In Table 1 I use the cumulative share to explore the decomposition of the additional government spending in response to to explore the decomposition of the detailed as  $\frac{\sum_{j=0}^{20} \partial \ln G_{i,t+j}}{\sum_{j=0}^{20} \partial \ln G_{t+j}} \frac{\bar{G}_i}{\bar{G}}$ , where  $G_{i,t+j}$  is the government spending in sector i at time t + j,  $G_t$  is total government spending,  $\overline{G}$ and  $G_i$  are the average total government spending and its components over the entire time series. The result show, for each one dollar of additional government spending during the "stimulus" period, 44 cents are spent in the government sector and 46 cents are used to stimulate the private economy. That is roughly half of the extra spending goes to the private sector and half goes to the government sector. This finding implies that there is potentially a significant employment reallocation effect caused by the increase in  $G_2$  that moves labor to the government sector from the private sector.





The solid lines are the impulse response functions (IRFs) and the dashed lines are the 95% confidence interval. The peak response of  $G_1$  is larger than the peak response of  $G_2$  as well as G, meaning that government spends more money in the private sector than it normally does.

e 1:	The dec	composition of a	additional government sper	1
-		Each one doll	ar of spending goes to:	
	Sector	Government	Private	
	Share	0.4368	0.4586	

Table ding

For each one dollar of additional government spending during the "stimulus" period, 44 cents are spent in the government sector which is roughly the same amount of money used to stimulate the private economy. This finding implies that there is potentially a significant employment reallocation effect caused by the increase in  $G_2$  that moves labor to the government sector from the private sector.

I then use the private sector output as the sixth variable in the VAR. Figure 3 shows the responses of GDP and private sector output. The responses are normalized such that the response of government spending is unity. This figure illustrates that the response of private sector output is less than GDP. The reason is that the overall output response includes the response of private sector output as well as the government sector output sector response. The government sector output is compensation of general government employees plus general government consumption of fixed capital. It is actually a component of government spending. The changes of this component should be treated as shocks rather than responses. Therefore, to examine the effects of government spending shocks on output, we should focus on the response of private output.





Figure 3: The solid line in the bottom panel is the GDP response and the dotted line is the private output response. This figure shows that the percentage deviation of private output is smaller than the GDP.

I calculate two types of government spending multipliers: the peak multiplier and the cumulative multiplier. The peak multiplier is calculated as  $|\frac{max_j\partial lnY_{i,t+j}}{max_j\partial lnG_{t+j}}\frac{\bar{Y}_i}{G}|sign(\frac{\partial lnY_{i,t+j}}{\partial lnG_{t+j}})$  for i = p or t, where  $Y_{i,t+j}$  is the output (value added) of private sector (denoted as p) or total output (denoted as t) at time t + j,  $G_t$  is government spending,  $\bar{G}$  and  $\bar{Y}_i$  are the average government spending and sector output over the entire time series. The cumulative multiplier, following Drautzburg and Uhlig (2011), is calculated as  $\frac{\sum_{j=0}^{20} \partial lnY_{i,t+j}}{\sum_{j=0}^{20} \partial lnG_{t+j}}\frac{\bar{Y}_i}{G}$ . Table 2 reports the government spending multipliers in private sector tor and total GDP. Although the total government spending multiplier is large, the multiplier of private sector output is much smaller. Therefore, the commonly used output multiplier overestimates the effects of government spending shocks on the private sector.

Fable	2: Governme	nt Spendi	ng Multipliers
_		Multipl	iers of:
		Private	Total
	Peak	0.6450	0.9471
	Cumulative	0.6074	1.1330

# 4 Government spending multipliers in a standard New Keynesian model

Since the empirical methods cannot isolate the responses of economic variables to a shock of a specific government spending component, I investigate the quantitative effects of each component of the government spending by the following method. I construct a standard New Keynesian model<sup>9</sup> with habit formation, sticky prices, capital adjustment costs and a distinction between government purchases of private and government output. Then, I estimate the model's parameters by matching the impulse responses of the model and the VAR. This step proves capability of the model in matching the data. Finally, I conduct counter-factual experiments by keeping one type of government spending shock each time to illustrate how the isolated shocks affect the economy. I also calculate the government spending multipliers for each component.

## 4.1 A standard New Keynesian model

There are two production sectors: a private sector, denoted as Sector 1, and a government sector, denoted as Sector 2. Households consumption comes from only the private sector, while government spend money on the goods and services from both sectors.

 $<sup>^{9}</sup>$ See Dupor et al. (2009).

#### 4.1.1 Households

A representative household maximizes its utility by choice of consumption  $C_t$ , nominal bonds  $B_t$ , and labor supply  $N_t$ . It maximizes:

$$\sum_{t=0}^{\infty} \beta^t E_0 \left[ \frac{(C_t - hX_{t-1})^{1-\sigma}}{1-\sigma} - \Psi_N \frac{N_t^{1+\theta}}{1+\theta} \right]$$
(13)

subject to:

$$C_t + I_t + \frac{B_t}{P_t R_t} = \frac{W_t}{P_t} N_t + r_t^k K_t + \frac{B_{t-1}}{P_t} + \frac{D_t}{P_t} + \frac{T_t}{P_t}$$
(14)

where,  $P_t$ ,  $W_t$ ,  $D_t$ , and  $T_t$  are the nominal prices of private sector goods, nominal wage level, profits of private firms and a lump sum tax/transfer from the government, respectively.  $C_t$ ,  $I_t$ , and  $K_t$  represent consumption, investment and capital stock, respectively.  $r_t^k$  is the real rental rate of capital. h is the parameter of habit persistence;  $\sigma$  is the inverse of intertemporal elasticity of substitution;  $\theta$  is the Frisch elasticity; and  $\Psi_N$  denotes the disutility of labor.

The term  $X_t$  is the level of habit accumulated by the household. Its law of motion is:

$$X_t = C_t + (1 - \delta) X_{t-1}$$
(15)

where  $\delta > 0$  is the depreciation rate of habit stock. In the above setting, let  $\beta = 0.99$ ,  $\sigma, \theta, \Psi_N > 0$ , and  $h, \delta \in (0, 1)$ .

The capital accumulation equation is

$$K_{t+1} = I_t + (1 - \delta_k)K_t - \frac{\sigma_I}{2}(\frac{I_t}{K_t} - \delta_k)^2 K_t$$
(16)

The parameter  $\sigma_I \geq 0$  governs the magnitude of capital adjustment costs and  $\delta_k$  is the depreciation rate of capital.

The household's problem is to maximize lifetime expected utility (13), subject to (14), (15), (16) and (3).

#### 4.1.2 Private Firms

In the private sector, there are final-good producers and a continuum of monopolistic competitive firms over [0, 1]. The problem of the final-good producers is the same as in Section 2. The discounted profits of the ith intermediate good producer are given by

$$E_t \sum_{j=0}^{\infty} \beta^{t+j} \lambda_{t+j} [P_{t+j}(i)y_{1t+j}(i) - (1-v)(W_{t+j}N_{1t+j}(i) + P_{t+j}r_{t+j}^k K_{t+j}(i))]$$
(17)

The production function for firm i's good is:

$$y_{1t}(i) = [K_t(i)]^{\alpha} [N_{1t}(i)]^{(1-\alpha)}$$
(18)

where  $N_{1t}(i)$  and  $K_t(i)$  denote the labor and capital input by the ith intermediate firm.

Each firm in the private sector optimally resets its price with probability 1 - v in each period. Let the optimally chosen price at t to be  $P_t^*$ . If a firm optimized its price at time t, then its time t + j price is given by:

$$P_{t,j}^* = P_t^* (\frac{P_{t+j-1}}{P_{t-1}})^{\gamma}$$
(19)

where,  $\gamma \in [0, 1]$  is the degree of price indexation. If a firm has the opportunity to optimize its price at t, it maximizes the expected discounted value of real profits (17) subject to the production function (18), the demand curve (5), and the Calvo-type price-setting friction.

#### 4.1.3 Government

The government in this model has two roles: a purchaser and a producer.

As a producer, it hires labors and produce outputs which are only purchased by the government. The production function of government sector is  $Y_{2t} = N_{2t}$ . Government purchases the output of this sector at cost,  $W_t N_{2t}$ .

As a purchaser, the government not only buys goods from the government sector, but also purchases goods and services from the private sector at the market price. All of the government spending is financed by the lump-sum taxes,  $P_tG_{1t} + P_{2t}G_{2t} = T_t$ .<sup>10</sup>

Following Edelberg et al. (1999), I assume that the log-deviation of  $G_{1t}$  and  $G_{2t}$  have finite ordered ARMA(p,q) representations, for i = 1, 2:

$$A_i(L)\hat{g}_{it} = B_i(L)\epsilon_{it} \tag{20}$$

where  $A_i(L)$  and  $B_i(L)$  are finite ordered polynomials in non-negative powers of the lag operator L. I assume  $A_i(L)$  have only roots that lie outside the unit circle, and  $\epsilon_{it}$  are iid shocks that are orthogonal to all model variables dated time t - 1 and earlier.

<sup>10</sup>note  $P_{2t} = W_t$ 

#### 4.1.4 Monetary Policy

The monetary policy is the same as in Section 2. As a reminder, the monetary authority uses the nominal interest rate,  $R_t$  as the instrument to conduct monetary policies. It determines  $R_t$  following a Taylor rule:

$$R_t = R_{t-1}^{\rho_r} \pi_t^{\phi_1(1-\rho_r)} (Y_{1t} + Y_{2t})^{\phi_2(1-\rho_r)}$$

where  $\rho_r \in (0, 1)$ . The monetary authority sets the nominal interest rate based on the last period's rate, inflation  $\pi_t$ , output of private sector  $Y_{1t}$ , and output of government sector  $Y_{2t}$ .

#### 4.1.5 Market Clearing

There are two goods market clearing conditions.

$$C_t + G_{1t} = Y_{1t}$$

and

$$G_{2t} = Y_{2t}$$

The labor market clearing conditions:

$$N_t = N_{1t} + N_{2t}$$
$$N_{1t} = \int_0^1 N_{1t}(i)di$$

and bonds market clearing.

#### 4.1.6 Equilibrium

A monetary equilibrium is a set of stochastic processes, {  $C_t$ ,  $I_t$ ,  $N_t$ ,  $W_t$ ,  $P_t$ ,  $Y_{1t}$ ,  $Y_{2t}$ ,  $N_{1t}$ ,  $N_{2t}$ ,  $K_t$ ,  $r_t^k$ ,  $R_t$ ,  $B_t$ ,  $T_t$ ,  $\pi_t$ ,  $P_t(i)$ ,  $y_{1t}(i)$ ,  $N_{1t}(i)$ ,  $K_t(i)$ ,  $v_t$ } such that for given { $G_{1t}$ ,  $G_{2t}$ } the households and firms problem are satisfied, the monetary and fiscal policy rules are satisfied, markets clear and the aggregate resource constraint holds.

### 4.1.7 Summary of Dynamic Equilibrium Model

The equilibrium of the model can be described by the following log-linearized equations.

$$\frac{-\sigma}{\delta-h}(\delta\hat{c}_t - h\hat{x}_{t-1}) - (1 - \frac{\beta h}{1 - \beta(1 - \delta)})\lambda_t - \frac{\beta h}{1 - \beta(1 - \delta)}\mu_t = 0$$
(21)

$$\frac{\sigma(1-\beta(1-\delta))}{\delta-h}(\delta\hat{c}_{t+1}-h\hat{x}_t) + \mu_t = \beta(1-\delta)\mu_{t+1}$$
(22)

$$\hat{x}_t = (1 - \delta)\hat{x}_{t-1} + \delta\hat{c}_t \tag{23}$$

$$r_t - E_t \pi_{t+1} = \lambda_t - E_t \lambda_{t+1} \tag{24}$$

$$\lambda_t = \varphi_t - \sigma_I \delta_k (\hat{i}_t - \hat{k}_t) \tag{25}$$

$$\hat{\varphi}_t = [1 - \beta(1 - \delta_k)](\lambda_{t+1} + \hat{r}_{t+1}^k) + \beta(1 - \delta_k)\varphi_{t+1} + \beta\sigma_I\delta_k^2(\hat{i}_{t+1} - \hat{k}_{t+1})$$
(26)

$$\hat{k}_{t+1} = \delta_k \hat{i}_t + (1 - \delta_k) \hat{k}_t$$
 (27)

$$\hat{y}_{1t} = \alpha \hat{k}_t + (1 - \alpha) \hat{n}_{1t}$$
 (28)

$$\pi_t - \gamma \pi_{t-1} = \beta (E_t \pi_{t+1} - \gamma \pi_t) + \kappa [(1 - \alpha)(\theta \hat{n}_t - \lambda_t) + \alpha \hat{r_t}^k]$$
(29)

$$\hat{r}_t^k = \theta \hat{n}_t - \lambda_t - \hat{k}_t + \hat{n}_{1t} \tag{30}$$

$$r_t = \rho_r r_{t-1} + (1 - \rho_r)(\phi_1 \pi_t + \phi_2(\frac{Y_1}{Y}\hat{y}_{1t} + \frac{Y_2}{Y}\hat{g}_{2t}))$$
(31)

$$\hat{n}_{2t} = \hat{g}_{2t} \tag{32}$$

$$\hat{n}_t = \frac{N_1}{N}\hat{n}_{1t} + \frac{N_2}{N}\hat{n}_{2t}$$
(33)

$$\hat{y}_{1t} = \frac{C}{Y_1}\hat{c}_t + \frac{I}{Y_1}\hat{i}_t + \frac{G_1}{Y_1}\hat{g}_{1t}$$
(34)

where  $\lambda_t$  is the multiplier on household's budget constraint;  $\mu_t$  is the multiplier on habit equation; and  $\varphi_t$  is the multiplier on the capital formation equation.  $\kappa \equiv \frac{(1-v)(1-\beta v)}{v}$ .

Eq. (21) is the first order condition with respect to current consumption. Eq. (22) is the first order condition with respect to  $x_t$ . Eq. (23) and (27) are the accumulations of habit stock and capital, respectively. Eq. (24) characterizes the optimal consumption-savings by households given the expectation of real interest rate. Eq. (25) and (26) represent the first order condition with respect to  $i_t$  and  $k_{t+1}$ , respectively. Eq. (28) is the production function of intermediate goods. Eq. (29) is the private firms optimal pricing behavior. Eq. (30) is the capital rental rate from intermediate good producer's cost minimization problem. The monetary policy is given by Eq. (31). The government sector production is given by (32). Labor market and goods market clearing conditions are (33) and (34), respectively.

### 4.2 Minimum distance estimation

In this subsection, I discuss the methodology for estimating and evaluating my model. I partition the model parameters into two groups. The first group contains  $\beta$ ,  $\alpha$ , and  $\delta_k$ .  $\beta$  is calibrated as 0.99, which corresponds to a steady-state annualized real interest rate of 4 percent. I set  $\alpha = 0.36$ , which implies a steady-state share of capital income roughly equal to 36 percent.  $\delta_k$  is set to be 0.02, which implies an annual capital depreciation rate of 10 percent. The second group of parameters is  $\theta_0 \equiv \{\sigma, \theta, \kappa, h, \delta, \gamma, \sigma_I, \rho_r, \phi_1, \phi_2\}$ , I estimate these parameters<sup>11</sup> by the minimum distance method. The minimum distance method is a well known method in the literature,<sup>12</sup> which provides the estimates by minimizing a measure of distance between the impulse responses generated by the economic model and those estimated. The objective function is defined as:

$$D = min_{\theta} [\gamma(\theta_0) - \gamma^s]' \Omega^{-1} [\gamma(\theta_0) - \gamma^s]$$

where  $\theta_0 = \{...\}$  is the "deep" parameter vector.  $\gamma^s$  is the impulse response function from the VAR model and  $\gamma(\theta_0)$  is the impulse response function from the theoretical model. I use the impulse responses of private fixed investment, private sector outputs, inflation and the 3-month T-bill rates as my targets. The exogenous shocks used in the model are the responses of  $G_{1t}$  and  $G_{2t}$  taken from the VAR.  $\Omega$  is the weighting matrix. Following Christiano et al. (2005), I set  $\Omega$  to be a diagonal matrix with the sample variance of  $\gamma^s$  on the main diagonal. The estimates,  $\hat{\theta}_0$ , is given by minimizing the objective function.

To simulate the model economy, I adopt Edelberg et al. (1999) parametrization of the government spending. That is:  $A_i(L) = 1$  and  $B_{ij}$  are the estimated response of real

<sup>&</sup>lt;sup>11</sup>I ignore the ZLB case in the estimation because it happens rarely in the history.

 $<sup>^{12}</sup>$ See, for example, Dupor et al. (2009).

government purchases at t + j to the defense news shocks at time t, for i = 1, 2 and j = 1, 2, ..., 20. The shocks are one time shocks to the two components of government spending at time t = 1 with a size of unity. The top two graphs of Figure 4 illustrate how the government spending respond to these shocks which are also depicted in Figure 3.

The solid lines in figure 4 reports the responses of investment, private sector output, inflation, and nominal interest rate to the defense news shock respectively, and the dashed lines are the 95% CI. The dotted lines are the impulse responses generated by the two sector model given the exogenous government spending series. In general, the IRFs from the model lie in the 95% CI of the IRFs from the data.

Table 3 provides the point estimates and standard deviations. Note that the slope of NKPC,  $\kappa$ , equals 0.0809 meaning that the price stickiness is not strong.



#### Figure 4: Matching the IRFs

The solid lines in figure 4 reports the responses of investment, private sector output, inflation, and nominal interest rate to the defense news shock respectively, and the dot lines are the 95% CI. The dash lines are the impulse responses generated by the two sector model given the exogenous government spending series. In general, the IRFs from the model lie in the 95% CI of the IRFs from the data.

Parameter	Meaning	DSGE estimation
$\hat{\sigma}$	Inverse of IES	$\begin{array}{c}1.4890\\\scriptscriptstyle(0.0046)\end{array}$
$\hat{ heta}$	Frisch elasticity	$\underset{(0.2650)}{2.0057}$
$\hat{\kappa}$	Slope of NKPC	0.0809 (0.0006)
ĥ	Habit persistence	$0.7826 \\ (0.0002)$
$\hat{\delta}$	Depreciation rate of habit stock	$0.4405 \\ (0.0002)$
$\hat{\gamma}$	Degree of price indexation	$0.4873 \\ {}_{(0.0004)}$
$\hat{ ho}_r$	Smoothing coefficient in Taylor rule	$0.8008 \\ (0.0009)$
$\phi_1$	Response to inflation in monetary policy	$\underset{(0.0004)}{1.2450}$
$\phi_2$	Response to output in monetary policy	$\substack{0.1472\\(0.0001)}$
$\hat{\sigma}_I$	Magnitude of adjustment costs	$\underset{(0.0061)}{19.8329}$

4.3 Quantitative effects of different components in government spending

Given the point estimations of the model, I therefore conduct counter-factual experiments to explore the quantitative effects of different components in government spending.

First, I shut down the shocks to  $G_{2t}$  and simulate the model using shocks to  $G_{1t}$ only. I then shut down the shocks of  $G_{1t}$  and simulate the model using shocks to  $G_{2t}$  only. Figure 5 illustrates the simulations. The dot-dash line is the response with the shock in  $G_{1t}$  only; the dash line is the response with the shock in  $G_{2t}$  only; the solid line is the response with two shocks simultaneously. The upper panels illustrate that the  $G_{1t}$  shock has positive effect on private sector output and hours worked, while the shock on  $G_{2t}$  has negative impact on private sector output and hours worked. In response to the negative wealth effect caused by an increase in either  $G_{1t}$  or  $G_{2t}$ , households increase  $N_t$  and decrease consumption. With shock in  $G_{1t}$  only,  $G_{2t}$  is fixed, and  $N_{2t}$  is constant. Therefore, the increase in  $N_t$  occurs through the increase in  $N_{1t}$ . Consequently,  $Y_{1t}$  increases. On the other hand, with shocks in  $G_{2t}$  only, the increase in total labor supply  $N_t$  occurs through the increase in  $N_{2t}$  and the decrease in  $N_{1t}$ . The reason is that the negative wealth effect requires the households to increase labor supply  $N_t$  but reduce consumption  $C_t$ . Because  $G_{1t}$ is fixed, the overall demand of private sector output  $Y_{1t} = C_t + G_{1t}$  falls. As a result,  $N_{1t}$  decreases. Therefore, the shocks to  $G_{2t}$  reallocates labor from the private sector to the government sector and lower the private sector output. The bottom panels show the responses of consumption and investment. An increase in  $G_{1t}$  leads to an increase in private consumption and investment, while a positive shock in  $G_{2t}$  leads to a decrease in private consumption and investment.

I also calculate the implied multipliers which is reported in Table 4. The multipliers show that one dollar increase in the government purchases of goods and services from private sector leads to a \$1.99 to \$2.52 increase in private sector output and a \$1.23 to \$1.44 increase in consumption. However, one dollar increase in the government purchases of goods and services from government sector results in a \$1.64 to \$1.81 lose of private sector output and a \$0.23 to \$0.25 lose of private consumption. Empirical evidence shows that a significant part of the unexpected changes in total government spending is used in the government sector output. Therefore, the overall multiplier of government spending shocks on private sector output is small. The policy implication is that to stimulate the private sector output, government needs to spend money in the private sector instead of the government sector.



Figure 5: effects of different components in government spending

The dot-dash lines are the responses with the shock in  $G_{1t}$  only; the dash lines are the responses with the shock in  $G_{2t}$  only; the blue solid lines are the responses with two shocks simultaneously. This figure shows that the  $G_{1t}$  shock has positive effect on private sector output, while the shock on  $G_{2t}$  has negative effect on private sector output.

rapid 4. Multipliers of different component in government spending						
	Private C	Output Multipliers of:	Consumption Multipliers of			
	$G_{1t}$	$G_{2t}$	$G_{1t}$	$G_{2t}$		
Peak	1.9884	-1.6419	1.2276	-0.2311		
Cumulative	2.5242	-1.8098	1.4415	-0.2468		

 Table 4: Multipliers of different component in government spending

## 4.4 Total government spending multiplier

I redistribute the series of  $G_t$  in the VAR between the private and government sectors. That is, denote  $\theta_g \in [0, 1]$  as the percentage of government spending shock spent in the private sector. I then simulate the model economy using  $G'_{1t} = \theta_g G_t$  and  $G'_{2t} = (1 - \theta_g)G_t$  as the shocks.

Figure 6 and Table 5 show the total government spending multipliers as a function of the percentage of government spending shock spent in the private sector. Both the peak and cumulative multipliers increase as more unexpected change of total government spending are spent in the private sector. This confirms the finding that the government spending shocks happens in the private sector has positive effect on the private output, while the effect of government spending shocks in the government sector is negative.



Figure 6: Total government spending multipliers on private sector

Figure 6 shows the total government spending multipliers as a function of the percentage of government spending shock spent in the private sector. The upper panel is the peak multiplier, and the bottom panel is the cumulative multiplier. The more spent in the private sector, the larger the total government spending multipliers on private sector.

_					
	percentage spent in private sector				
$0 \qquad 0.25 \qquad 0.5 \qquad 0.75 \qquad 1$					
Peak	-1.3272	-0.4907	0.3458	1.1823	2.0188
Cumulative	-1.8384	-0.7585	0.3215	1.4014	2.4813

Table 5: Total government spending multipliers on private sector

# 5 Reconciling Conflicts

In the literature, there is no consensus regarding the effects of government spending shocks on private consumption. The SVAR approach<sup>13</sup> developed by Blanchard and Perotti (2002) finds that a positive government spending shock raises consumption. In contrast, the narrative approaches developed in Ramey and Shapiro (1998) and Ramey (2011) typically find that government spending lowers consumption. Based on the above findings, this paper can potentially reconcile the conflicts regarding the response of private consumption among different identification strategies. That is, the response of consumption varies with the structure of the identified government spending shock. If a method identifies a government spending shock with more money spent in the private sector, it tends to provide a positive impact on consumption, and vice versa. In this section, I provide empirical evidence to support this finding.

I conduct three VAR exercises using different identification methods: the SVAR approach, the Ramey and Shapiro (1998) war dates approach, and the Ramey (2011) defense news approach. The variables in the VAR are the same as in Ramey (2011)Section III.C: real per capita government spending, real per capita GDP, total hours worked, real non-durable and service consumption, real private fixed investment, the Barro and Redlick (2011) average marginal income tax rate, and nominal compensation in private business divided by the deflator in private business. All variables are in log term and the data is taken from Ramey (2011). To see how the structure of the identified government spending shocks varies with different identification strategies, I run another VAR with only the exogenous shock variable, total government expenditure and total government hours worked including armed force.<sup>14</sup> Figure 7 compares the impulse responses from the VARs. The responses are normalized such that the total government spending's peak response is equal to one. As shown, in response to a defense related shock, the government increases more government hours worked; while the SVAR approach identifies a much less increase in government hours. In other words, the SVAR approach identifies a government spending shock with more

<sup>&</sup>lt;sup>13</sup>The SVAR approach is a widely used method in studying the government spending shocks. Blanchard and Perotti (2002) find that government spending does not contemporaneously respond to total output. Thus, the identification of government spending shocks is identical to a Choleski decomposition where government spending is ordered first.

<sup>&</sup>lt;sup>14</sup>The government hours worked data is the Francis-Ramey Updates taken from Ramey's website. For the SVAR approach, only total government expenditure and hours worked variables are included in the VAR.

money spent in the private sector, while the narrative approaches identify government spending shocks with more expenditure in the government sector. This result is nature. The narrative approach shows how the government responds to a (or a potential) war, hence it is nature to see the government increase its hours worked, especially military hours, during the war time. On the other hand, the consumption response is positive with the SVAR approach; it is around zero with the Defense News approach; and it is negative with the War Dates approach. This result is consistent with the findings in the above sections. That is, an increase in government purchases in the private sector can lead to an increase in private consumption, while an increase in government purchases in the government sector will lower private consumption. In sum, since the narrative approaches identify government spending shocks with more expenditure in the government sector, they typically find a negative response of private consumption. While the SVAR approach identifies a government spending shock with less money spent in the government sector, it finds an increase in private consumption.



Figure 7: Comparison between Svar and the Narrative Approaches

Figure 7 compares the impulse responses identified by the SVAR approach and the Narrative approaches (with defense news series or war dates). The middle panel shows that the narrative identification approaches identify government spending shocks with larger increases in government hours. The bottom panel shows that the private consumption increases less with more increase in government hours.

Table 6 also provides the peak and cumulative consumption multipliers of these three approaches. It confirms that the private consumption multipliers become smaller when the government hours worked variable increase more.

	Table 6: Identification and Multipliers					
	Gov hours elasticity			Private consumption multiplier		
	Svar	Defense News	War Dates	Svar	Defense News	War Dates
Peak	0.4945	1.0041	1.1429	0.1211	0.0185	-0.0146
Cumulative	0.5142	1.0296	1.1673	0.0283	-0.1338	-0.4556

# 6 Conclusion

This paper study the effects of shocks to different components in government spending. Empirical evidences show that the commonly used output multiplier overestimates the effects of government spending shocks on the private sector. Shocks to the two components of government spending have opposite effects on the private sector. A one dollar increase in the government purchases of goods and services from the private sector leads to a \$1.99 to \$2.52 increase in the private sector output. However, a one dollar increase in the government purchases of goods and services from the government sector results in a \$1.64 to \$1.81 loss of the private sector output. Moreover, this paper can potentially reconcile the conflicts regarding the consumption responses among different identification strategies. That is, the response of consumption varies with the structure of the identified government spending shocks. The policy implication is that to stimulate the private sector output, the government needs to spend money in the private sector instead of the government sector.

# Appendix

In this appendix, I discuss what are the effects of these two shocks on marginal costs. In this model, marginal cost equals to wage. We can solve:

$$\hat{mc}_{t} = \hat{w}_{t} = \left[\frac{\sigma(\alpha_{1} - g_{1})}{1 - g_{1}} + \theta\alpha_{1}\frac{Y_{1}}{Y}\right]\hat{g}_{1t} + \left[\frac{\sigma\alpha_{2}}{1 - g_{1}} + \theta(\alpha_{2}\frac{Y_{1}}{Y} + \frac{Y_{2}}{Y})\right]\hat{g}_{2t}$$

A shock to  $G_{1t}$  affects the marginal cost through two channels. First, it increases labor through the second term,  $\theta \alpha_1 \frac{Y_1}{Y} \hat{g}_{1t}$ . The increase of labor demand puts upward pressure on the marginal cost. Second, it changes households consumption through  $\frac{\sigma(\alpha_1-g_1)}{1-g_1}\hat{g}_{1t}$ . This is actually the response of consumption to both negative wealth effect, and the New Keynesian mark-up effect. Overall, the negative wealth effect dominates, so consumption decreases and puts downward pressure on the marginal cost.

Similarly, a shock to  $G_{2t}$  affects the marginal cost through the labor and consumption channels as well. In this case, the negative wealth effect again dominates, so consumption drops. However, the sign of the response of equilibrium labor  $\theta(\alpha_2 \frac{Y_1}{Y} + \frac{Y_2}{Y})\hat{g}_{2t}$  is ambiguous. The shocks to  $G_{2t}$  asks for more labor demand in the government sector, while it also reallocates some labor from private sector to government sector. The overall effects on total labor depends on the size of  $\alpha_2 \frac{Y_1}{Y}$  and  $\frac{Y_2}{Y}$ .

# References

- Barro, Robert J and Charles J Redlick, "Macroeconomic effects from government purchases and taxes," *The Quarterly Journal of Economics*, 2011, 126 (1), 51–102.
- Blanchard, Olivier and Roberto Perotti, "An empirical characterization of the dynamic effects of changes in government spending and taxes on output," the Quarterly Journal of economics, 2002, 117 (4), 1329–1368.
- Burnside, Craig, Martin Eichenbaum, and Jonas D.M. Fisher, "Fiscal shocks and their consequences," Journal of Economic Theory, 2004, 115 (1), 89 117.
- Cavallo, M., "Government employment and the dynamic effects of fiscal policy shocks," *Federal Reserve Bank of San Francisco Working Paper Series*, 2005, (16).
- Christiano, L., M. Eichenbaum, and S. Rebelo, "When is the government spending multiplier large?," *Journal of Political Economy*, 2011, 119 (1), pp. 78–121.
- Christiano, L.J., M. Eichenbaum, and C. Evans, "Nominal rigidities and the dynamic effects of a shock to monetary policy," *Journal of Political Economy*, 2005, 113 (1), 1–45.
- **Drautzburg, T. and H. Uhlig**, "Fiscal stimulus and distortionary taxation," Technical Report, National Bureau of Economic Research 2011.
- Dupor, Bill, Jing Han, and Yi-Chan Tsai, "What do technology shocks tell us about the New Keynesian paradigm?," *Journal of Monetary Economics*, 2009, 56 (4), 560–569.
- Edelberg, Wendy, Martin Eichenbaum, and Jonas DM Fisher, "Understanding the effects of a shock to government purchases," *Review of Economic Dynamics*, 1999, 2 (1), 166–206.
- Finn, M.G., "Cyclical effects of government's employment and goods purchases," International Economic Review, 1998, pp. 635–657.

- Mountford, Andrew and Harald Uhlig, "What are the effects of fiscal policy shocks?," Journal of applied econometrics, 2009, 24 (6), 960–992.
- Ramey, Valerie A, "Identifying Government Spending Shocks: It's all in the Timing\*," The Quarterly Journal of Economics, 2011, 126 (1), 1–50.
- and Matthew D Shapiro, "Costly capital reallocation and the effects of government spending," in "Carnegie-Rochester Conference Series on Public Policy," Vol. 48 Elsevier 1998, pp. 145–194.
- Rossi, Barbara and Sarah Zubairy, "What Is the Importance of Monetary and Fiscal Shocks in Explaining US Macroeconomic Fluctuations?," *Journal of Money, Credit and Banking*, 2011, 43 (6), 1247–1270.